THE APPROXIMATION OF JOINT DISTRIBUTION FUNCTIONS FOR APPLICATION IN PROBABILISTIC MECHANICAL DESIGN

by

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CHAPTER I

INTRODUCTION

Design engineers have begun to appreciate the limitations of the traditional or classical method of design. For example, the classical criteria for strength-limited design is that the overall strength of the system must be greater than the maximum applied load to the system multiplied by a factor of safety. This safety factor is usually a value between 1.3 and 6, depending on the structure and the materials. Although this approach is very simple in application, it often leads to designs that are overdesigned in terms of weight and bulk. Haugen [1] points out that "Since safety factors are not a performance-related measure there is no way by which an engineer can know whether his designs are near optimum or overconservative."

Designers are now starting to realize that applied loads, dimensions, material properties, etc. are all subject to variation. From this realization has emerged

a new design method called Probabilistic Design. Probabilistic Design treats the design inputs as random variables with each random variable having its own probability density function. After the variables have been defined, the problem can be solved using probability mathematics. The results can then be analyzed to determine if the design satisfies the operational and economic requirements with an acceptable reliability of success.

Although formulating the functional relationships of the random variables into a joint probability integral form is usually possible, evaluating this integral can be a very complex mathematical operation. Springer [2] has written an entire book on the subject of algebraic operations of random variables. Many of the chapters of his book deal with deriving the joint distribution and joint density functions of rather simple functions which contain random variables. Although the mathematical techniques that he describes are certainly important from a theoretical standpoint, there are some disadvantages in practical applications. The designer needs to have a working knowledge of differential and integral calculus, statistical inference, and probability distribution theory. Another drawback is that there are many

functions for which it is impossible to obtain the exact solution to the probability density function. Many times the system is so involved and cumbersome that the average engineer does not have the time or the knowledge to solve the problem.

An alternative to the traditional mathematical approach to Probabilistic Design is a technique known as Monte Carlo Simulation. Law and Kelton [3] define Monte Carlo simulation as "...a scheme employing random numbers which is used for solving certain stochastic or deterministic problems where the passage of time plays no substantive role". The random numbers are generated so that they simulate the physical random process of the probabilistic problem. The system is simulated many times so the frequencies of the events can be studied.

Monte Carlo [4] simulation originated as a tool for the development of the atomic bomb during World War II. The scientists used Monte Carlo simulation to simulate the randomness of neutron diffusion in fissle material. In the last thirty years Monte Carlo techniques have been used for operational research and nuclear physics. Since the motions of neutrons are random, the nuclear physicists have used Monte Carlo to simulate the performance of nuclear reactors. The physicists can

simulate the reactor design without the dangers and expense of an actual physical experiment.

Although the Monte Carlo technique has been around for over 50 years, its application in mechanical engineering design has been limited. The purpose of this thesis is to develop a simulation algorithm that is capable of approximating the joint probability distribution function of complex algebraic expressions encountered in design engineering. This computer algorithm is similar to the Monte Carlo method of simulation. However, this new technique does not use generated random numbers to represent the probability density functions of the individual random variables. Instead, the program divides each density function into a given number of discrete sections with equal probabilities and then takes all possible outcomes into consideration before assembling the joint distribution and density curves.

The literature review that follows is an overall review of probabilistic design and how simulation has been used in different engineering applications.

CHAPTER II

LITERATURE REVIEW

Probabilistic Machine Design

Bury [5] compared the probabilistic machine design approach to the traditional design method of a simple tension link with a given safety factor, material property, applied load, and cross sectional area. Bury pointed out that many design engineers are dissatisfied [6] with the traditional method because applied loads, material properties, and actual machining dimensions can vary dramatically from one application to another. While the traditional method assumes the input variables to be deterministic, the probabilistic method assumes the variables to be random with unique probability density functions. Once the density functions have been determined, the problem can then be solved as a probabilistic event. However, he points out that this probabilistic approach can only be taken if data is available to properly describe the random variables.

Balkey, Meyer, and Witt [7] discussed the importance of using Probability Structural Mechanics as a tool for evaluating the reliability and structural risk of components and structures. Probability Structural Mechanics, or PMS, combines the traditional methods of structural mechanics with the probabilistic methods to determine the probability of structural failure. They pointed out the necessity of including uncertainties in the mechanical and structural design process. They concluded that "...as components and structures become more sophisticated, failure mechanisms will be probabilistically modeled from the beginning of the design process, and potential design improvements will be evaluated to assess their effects on reducing overall risk."

Monte Carlo Simulation

Haugen [1] discussed Monte Carlo simulation throughout his text. One rather simple example was a simulation of strain energy in a statically loaded wire. Haugen solved the problem by Monte Carlo simulation and by an approximate partial derivative method and compared the results. After 200 simulations the strain energy

values were plotted on normal and lognormal probability paper.

Elishakoff [8] used Monte Carlo simulation to predict the buckling time of an elastic bar. The initial imperfections of the bar were assumed to be normally distributed with a given mean and autocorrelation function. The problem was then simulated many times to determine the reliability of the structure. A histogram and distribution of the buckling time was plotted. The results were then compared to the previously published works of Linberg [9]. The Monte Carlo solution was very similar to the results obtained by Linberg.

Elishakoff continued his work of simulating structural imperfections using the Monte Carlo method. His later paper [10] discussed buckling of finite uniform columns resting on a "softening" nonlinear elastic foundation. The initial imperfections were again assumed to be normally distributed. The system was simulated many times and the reliability was plotted vs. nondimensional actual load. The nondimensional buckling loads were then plotted as a histogram and analyzed.

Elishakoff's latest research [11] was concerned with the effect of nonsymmetric random imperfections on the reliability of axially compressed cylindrical shells. He

used the Monte Carlo process to obtain the reliability functions of shells with asymmetric and axisymmetric imperfections. Using the results of his previous works, Elishakoff was able to use measured initial imperfections (variance-covariance matrices and the mean vectors) as a direct input for the Monte Carlo simulation.

Martin [12] developed vectorized and scalar particle transport Monte Carlo algorithms to simulate the transport of photons in a high-density, high-temperature plasma. The behavior of a photon was simulated by drawing random samples from probability distributions that describe the actual physical process. The results were tabulated to chart the overall tallies. Both algorithms were compared to a reference Monte Carlo code from Lawrence Livermore National Laboratory.

Ramsay [13] studied the variability of deflections of ten reinforced concrete fixed-ended T-beams and slabs. The dimensions of the beams, as well as the material properties and type of loadings were assumed to be random variables. A mathematical model was developed to describe the characteristics of the beams and slabs. The model considered the change of stiffness and the redistribution of moment along the length of the structure. Each of the structures was analyzed for four

different loading conditions. For each loading condition, 500 deflection values were generated using Monte Carlo simulation. These values were then converted to deflection ratios and statistically analyzed. The results of the simulation were compared to the deflections of those computed using the ACI 318-63 and ACI 318-71 Code procedures [14,15]. He concluded that the major causes of variability in deflections are the variability of the beam stiffness and the variability of the concrete strength.

Dao-Thien and Massoud [16] discussed the probabilistic distributions of stress and strength relationships. They presented a detailed mathematical background of a method used to obtain the stress and strength distribution functions. The density and distribution functions were written in the form of multiple integrals. A computer program was developed that assists the engineer in selecting a probability distribution (normal, log-normal, or Weibull) using linear regression. The program picks the distribution that minimizes the error function. An example was presented that illustrated a typical structural problem in the aerospace industry. The system was simulated 20,000 times using generated random numbers. The normal,

log-normal, and Weibull distributions were compared to the generated stress and strength data. The program picked the Weibull distribution as the best approximation of the design stress and the normal distribution for the design strength. The two distributions were compared to the histogram frequency distributions generated by the Monte Carlo method.

Fast Probability Integration

Hasofer-Lind [17], Rackwitz-Fiessler [18], Chen-Lind [19], and Wu [20] have developed a technique called Fast Probability Integration (FPI). The FPI method [21] assumes that all of the random variables are independent with known probability density functions. The random variable's density functions are transformed into equivalent normal random variables using a curve-fitting routine. The design point is then approximated using the scheme of normal tail approximation [22]. Wirsching and Wu [23] have tested various combinations of linear and nonlinear functions containing both normal and non-normal random variables. They report that the FPI method is more accurate and approximately 10 to 100 times faster than the Monte Carlo technique for approximating probabilities

of complicated functions involving several random variables. They point out that while the Monte Carlo method may be useful as a research tool, the large amount of computer processing unit time needed to approximate low values of probability tends to make the Monte Carlo method expensive and impractical.

Discrete Simulation

Lockwood [24] introduced a new method of computing the probability density functions in turbulent flows. Lockwood pointed out that the traditional method of probabilistic design has several shortcomings. The primary disadvantage is that the "...computational burden increases more or less exponentially with dimensionality." He added that he has developed a more economical approach to the solution of the probability density function of the transport equation. He applied his method to a fully turbulent jet having a given inlet pipe Reynolds number and temperature. He assumed the temperature to be a random variable. To calculate the temperature's probability density function, he divided the range into a finite number of discrete points. The various relations were applied to each discrete point. The total solution

was obtained by summing each discrete point. He compared the predicted probability density functions with the measured probability density functions and determined that the agreement was remarkably good. He concluded that his new method has an advantage over the traditional Monte Carlo simulation technique. Since the discrete method does not use generated random numbers, there is no statistical errors associated with the generation of the random numbers, as is the case with the Monte Carlo technique.

In this study, the probability density functions are divided into discrete points similar to the study conducted by Lockwood. However, this paper goes on to approximate the joint distribution function of algebraic expressions containing several independent random variables with known density functions. The methods and procedures for this simulation are discussed in Chapter 3. The computer programs that are used to approximate the joint probability distribution and density functions are described in Chapter 4. In Chapter 5 the discrete approximation is compared to the traditional Monte Carlo technique for a joint probability distribution that can be solved exactly. Chapter 6 contains an example of how this probabilistic method can

be used as a tool by engineers to aid in the design and development of structures and components. In Chapter 7 a different technique is discussed that determines the upper and lower bounds of a joint probability distribution using the discrete approximation. The conclusions and recommendations for further study are given in Chapters 8 and 9 respectively.

CHAPTER III

METHOD AND PROCEDURE

The purpose of this thesis is to develop a simulation algorithm that is capable of approximating the joint probability distribution function of complex algebraic expressions containing several independent random variables with known density functions. If the complex algebraic expression is represented by a variable Z, then Z can be written as

$$Z = Z(X_1, X_2, \ldots . X_n) \qquad (3.1)$$

where $X_1, X_2, \ldots . X_n$ are continuous, independent random variables with individual probability density functions of $f_1(x_1)$, $f_2(x_2)$, $\ldots . f_n(x_n)$ respectively. Since the variables in this study are assumed to be independent, the joint probability density function of Z can be written as the product of the X random variables' density functions.

$$f(z) = f_1(x_1)f_2(x_2).....f_n(x_n) \qquad (3.2)$$

The joint probability distribution function of the random variable for any n-dimensional set of Z can be written as,

$$F(z) = \int\int..... \int_{Z} f_1(x_1)f_2(x_2).....f_n(x_n)dx_n.....dx_2dx_1 \qquad (3.3)$$

If the random variables are bounded by the intervals (a_1, b_1), (a_2, b_2),.....,(a_n, b_n), the probability

$$P(a_1 \leq X_1 \leq b_1,\ a_2 \leq X_2 \leq b_2,\ .....,\ a_n \leq X_n \leq b_n) =$$

$$\int_{a_1}^{b_1} \int_{a_2}^{b_2} ..... \int_{a_n}^{b_n} f(x_1,x_2,.....,x_n)dx_n.....dx_2dx_1 \qquad (3.4)$$

Although formulating the density functions into a multiple integral form similar to equations 3.3 and 3.4 is usually possible, integrating the function over the range of the Z event can be a very complex mathematical operation.

In this chapter, a method that approximates the joint probability distribution function is discussed. The method, called discrete simulation, has four steps:

1. Cell Development
2. Discrete Point Representation
3. Simulation
4. Distribution Approximation

CELL DEVELOPMENT

In order to approximate the continuous density function of each random variable, the known density functions are divided into n sections or cells. Each cell is computed to have equal area, thus equal probability of occurrence.

$$1.\quad \int_{a_o}^{a_1} f(x)\,dx = \frac{1}{n} \qquad (3.5)$$

$$2.\quad \int_{a_1}^{a_2} f(x)\,dx = \frac{1}{n}$$

$$\vdots \qquad\qquad \vdots$$

$$n.\quad \int_{a_{n-1}}^{a_n} f(x)\,dx = \frac{1}{n}$$

Figure 3.1 shows an example of a normal random variable density function divided into seven cells with each cell having equal area. For sake of illustration, the density function has been divided into only seven cells. For practical applications the density function can contain anywhere from 30 to 200 cells.

DISCRETE POINT REPRESENTATION

After the function has been divided into equal areas, each cell is represented by a single discrete point. This point is located at the cell's center of gravity where:

$$\int_{a_o}^{a_{cg}} f(x)\,dx = \int_{a_{cg}}^{a_1} f(x)\,dx = \frac{1}{2n} \qquad (3.6)$$

and has a magnitude of 1/n. Figure 3.2 shows a normal density function divided into seven cells with its center of gravity locations. Table 3.1 lists the center of gravity locations of a normal distribution approximation for seven cells.

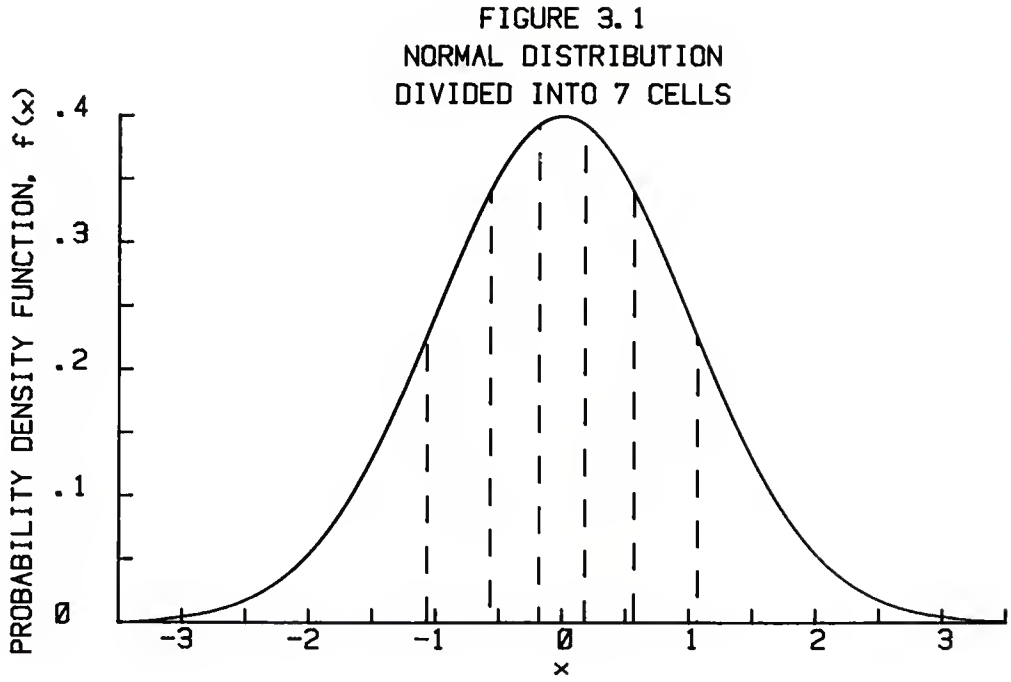


FIGURE 3.1
NORMAL DISTRIBUTION
DIVIDED INTO 7 CELLS
PROBABILITY DENSITY FUNCTION, f(x)
.4
.3
.2
.1
Ø
-3
-2
-1
Ø
1
2
3
x

FIGURE 3.2
7 CELL DISCRETE APPROXIMATION
OF A NORMAL DISTRIBUTION

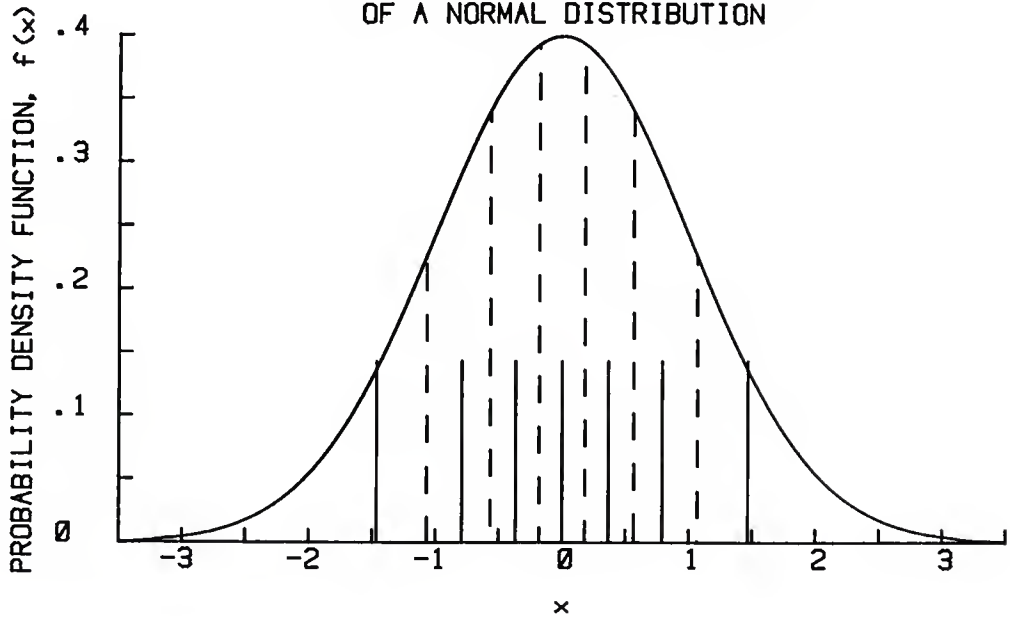


Table 3.1

Normal Distribution Approximation
Using Seven Discrete Points

Distribution: Normal
Mean: 0.0
Variance: 1.0
Standard Deviation: 1.0
Number of Cells: 7

Cell Number	Center of Gravity Location
1	-1.4655158
2	-0.7914320
3	-0.3656621
4	0.0000000
5	0.3656621
6	0.7914320
7	1.4655158

SIMULATION

Once each variable's density function has been chosen, all possible combinations of the expression are simulated. For example, if the joint probability distribution function of the equation

$$Z = A * B$$

is desired, and if the density functions of A and B are simulated using 100 and 150 discrete points respectively. The total possible combinations of Z is 100 * 150 = 15,000.

DISTRIBUTION FUNCTION

After the outcomes Z, have been computed, the data is sorted from smallest to largest. Each outcome has a probability of 1/m, where m is the number of total outcomes. The joint cumulative distribution function can then be assembled by summing the ordered value's probabilities.

$$F(z) = \begin{cases} 0 & i<Z_1 \\ F(z_{i-1}) + \frac{1}{m} & Z_1 \leq z < Z_{i+1} \text{ for } i=1,2,\ldots m-1 \\ 1 & Z_m \leq z \end{cases} \quad (3.7)$$

If the values between the discrete points need to be known, a linear empirical distribution function can be written that interpolates between the known points:

$$G(z) = \begin{cases} 0 & \text{if } z<Z_o \\ G(z_{i-1}) + \frac{z-Z_{i-1}}{Z_i - Z_{i-1}} G(z_i)-G(z_{i-1}) & \text{if } Z_{i-1} \leq z < Z_i \\ 1 & \text{if } Z_m \leq z \end{cases} \quad (3.8)$$

The computer program that performs the distribution approximation is listed in the Appendix.

DENSITY FUNCTIONS

Although developing a joint density function is not the primary purpose of this paper, two methods are used with reasonable success. These methods are only used to give an approximate shape of the joint probability density function and not an exact solution. The fitting of density functions is a complex problem in itself. The two methods are: histograms and cubic splines.

Histograms:

A histogram is simply a graphical estimate of the density function corresponding to the known data. To construct a histogram, the range of data is divided into k equal intervals. The data is then placed in its appropriate interval. Once all of the data has been sorted, the proportion of the values in each interval is determined.

The histogram is constructed using Sturge's [25] rule. The number of cells k is determined by the equation

$$k = 1 + 3.3 * \log_{10} m \qquad (3.9)$$

The range r, is estimated by the relationship

$$r = x_{max} - x_{min} \qquad (3.10)$$

And the width w, of each interval in the histogram is

$$w = r/k \qquad (3.11)$$

Although the histogram is easy to construct, the main disadvantage is the loss of information from grouping the data. Because of this, the histogram is used only as a reference to the possible density function's shape.

Cubic Splines:

A cubic spline is a set of cubic polynomials which is joined together to form a continuous function. A cubic spline subroutine is used to compute the first derivative of the function. The subroutine also determines a "smoother" approximation of the distribution function. The spline method uses a cubic interpolating function to approximate a "smooth" line through a given set of data points by minimizing the integral of the second derivative squared of the cubic function. A tolerance is imputed that allows the programmer to

tighten or loosen the curve fit. A detailed description of cubic splines can be found in reference [26].

The computer program listed in Appendix B performs the density approximations outlined above.

EXAMPLE

Given: Function Z=(X*Y**3)/12, where X and Y are continuous independent random variables. X is a uniform random variable with given probability density function:

$$f(x) = \begin{cases} 1/3 & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Y is a triangular random variable with a given probability density function:

$$f(y) = \begin{cases} \frac{2(y-4)}{7} & 1 \leq y \leq 2.5 \\ \frac{2(4-y)}{5.25} & 2.5 < y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find: The joint probability distribution of random variable Z=Z(X,Y).

For simplicity each cell is approximated using only five cells. The area of each cell is equal to 1/5. The

bound for the first cell of variable X is found by using the relationship:

$$\int_{a_o}^{a_1} f(x)dx = \text{Area of the cell}$$

$$\int_{1}^{a_1} 1/3dx = 1/5$$

$$x/3\Big|_{1}^{a_1} = 1/5$$

$$a_1 = 8/5$$

The center of gravity for cell 1 is computed using equation 3.6

$$\int_{1}^{a_{cg}} 1/3dx = 1/10$$

$$a_{cg} = 13/10$$

The remaining locations can be found using the same method. Figures 3.3 and 3.4 show the two density functions and their respective centers of gravity. Table 3.2 lists the center of gravity locations for both random variables.

FIGURE 3.3
5 CELL DISCRETE APPROXIMATION
OF A UNIFORM DISTRIBUTION

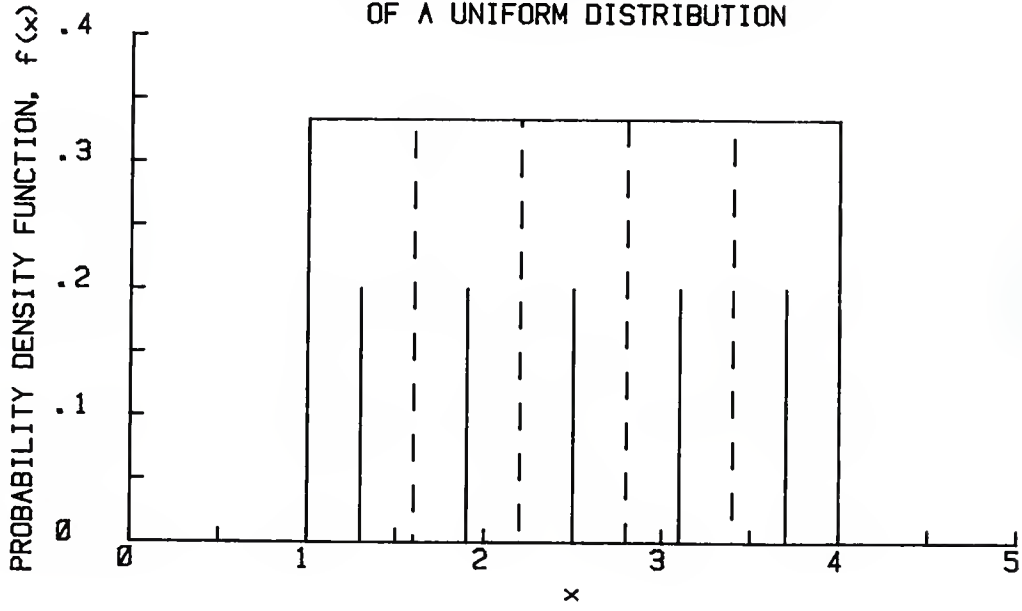


FIGURE 3.4
5 CELL DISCRETE APPROXIMATION
OF A TRIANGULAR DISTRIBUTION

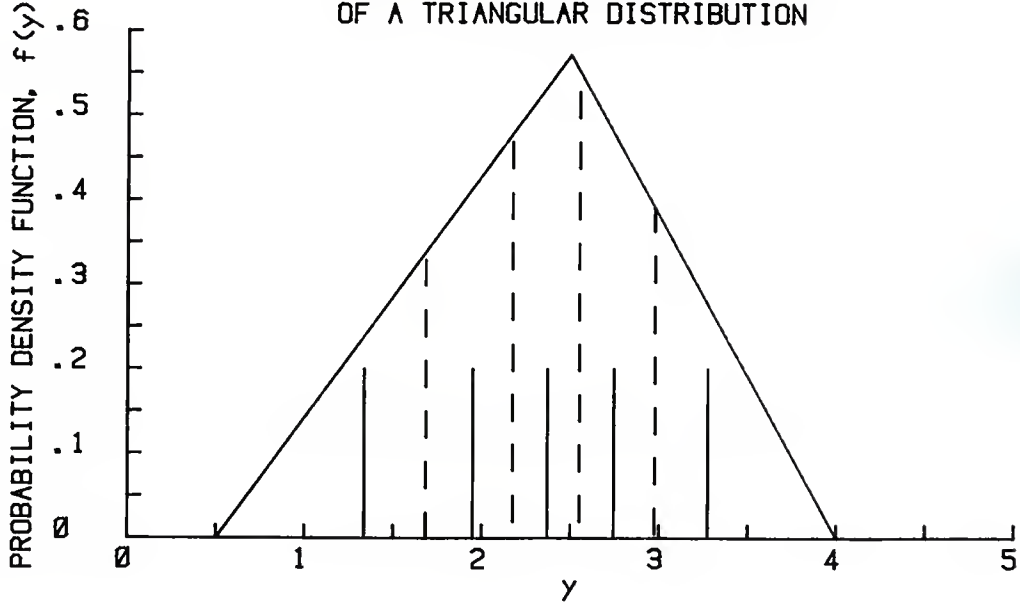


Table 3.2

Discrete Approximation
of Random Variables X and Y
Using Five Cells

Random Variable: X	Random Variable: Y
Distribution: Uniform	Distribution: Triangular
Mean: 2.5	Mean: 2.3333
Variance: 0.75	Variance: 0.5139
Number of Cells: 5	Number of Cells: 5

Center of Gravity Location	Cell Number	Center of Gravity Location
1.300000	1	1.336660
1.900000	2	1.949138
2.500000	3	2.370829
3.100000	4	2.745010
3.700000	5	3.275431

After computing all possible values, the 25 unique Z locations are sorted. Each Z location has a probability of 1/25. Figure 3.5 shows the 25 discrete Z locations. Using equations 3.7 and 3.8 a joint probability distribution function is assembled. Table 3.3 shows the sorted Z locations and their distribution values. Figure 3.6 shows the assembled Z distribution function.

Figures 3.7 and 3.8 show the Z distributions of the spline smoothing subroutine for a tolerance of 1.0 and 1.5 respectively.

FIGURE 3.5
Z DISTRIBUTION OF 25 DISCRETE
LOCATIONS Z=(X*Y^3)/12

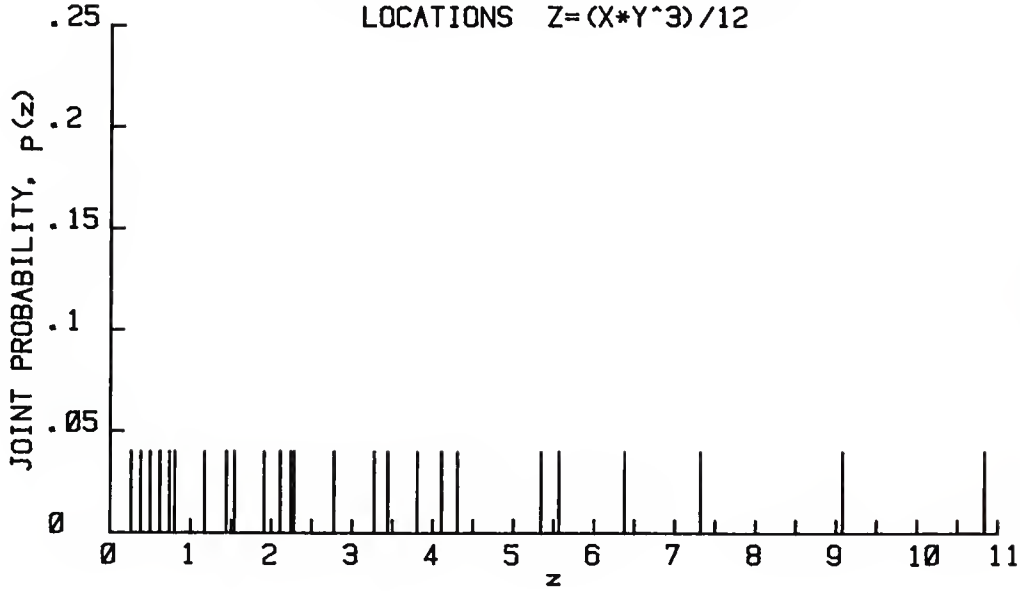


Table 3.3

Distribution of 25 Discrete
Z Locations
Z=(X*Y**2)/12

Number	Z Location	Z Distribution
1	0.2587170	0.0400000
2	0.3781249	0.0800000
3	0.4975327	0.1200000
4	0.6169406	0.1600000
5	0.7363484	0.2000000
6	0.8022129	0.2400000
7	1.1724650	0.2800000
8	1.4436524	0.3200000
9	1.5427172	0.3600000
10	1.9129693	0.4000000
11	2.1099535	0.4400000
12	2.2407525	0.4800000
13	2.2832214	0.5200000
14	2.7762546	0.5600000
15	3.2749459	0.6000000
16	3.4425557	0.6400000
17	3.8068655	0.6800000
18	4.1088568	0.7200000
19	4.3091393	0.7600000
20	5.3433328	0.8000000
21	5.5638804	0.8400000
22	6.3775262	0.8800000
23	7.3208952	0.9200000
24	9.0779101	0.9600000
25	10.8349250	1.0000000

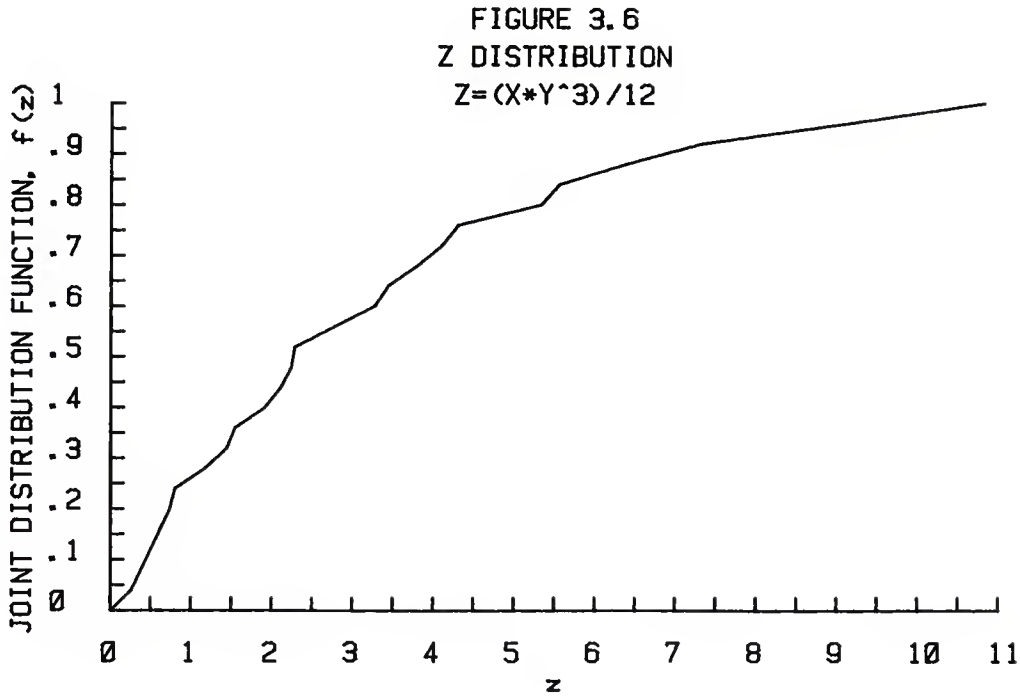


FIGURE 3.6
Z DISTRIBUTION
Z=(X*Y^3)/12
JOINT DISTRIBUTION FUNCTION, f(z)
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0 1 2 3 4 5 6 7 8 9 10 11
z

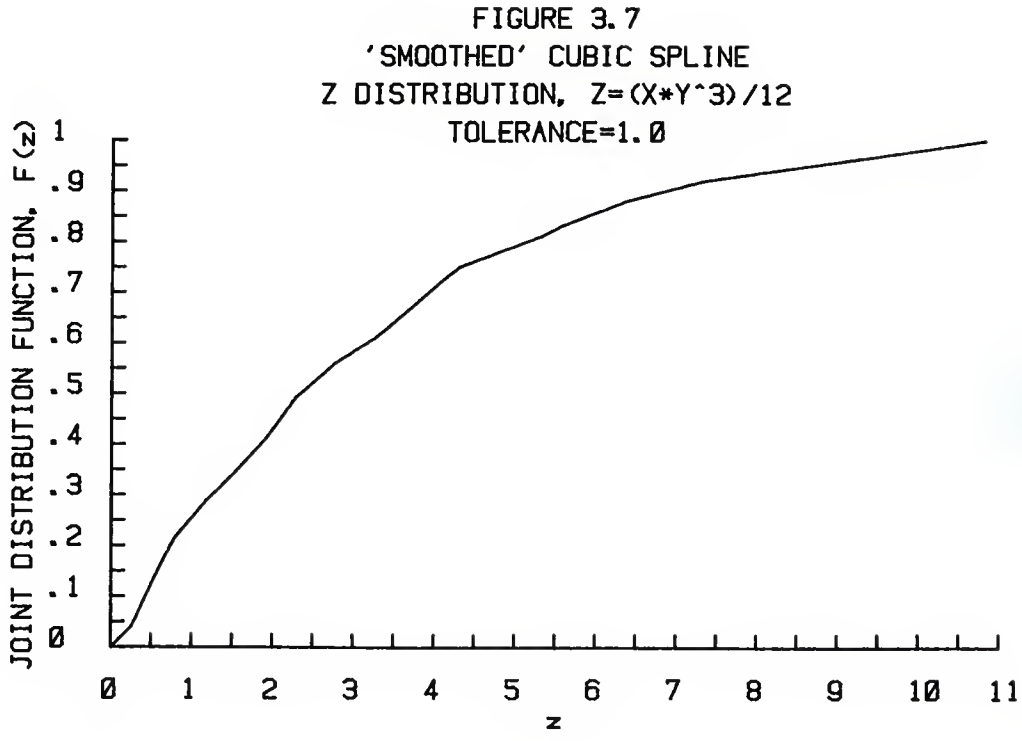


FIGURE 3.7
'SMOOTHED' CUBIC SPLINE
Z DISTRIBUTION, Z=(X*Y^3)/12
TOLERANCE=1.0
JOINT DISTRIBUTION FUNCTION, F(z)
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
6
7
8
9
10
11
z

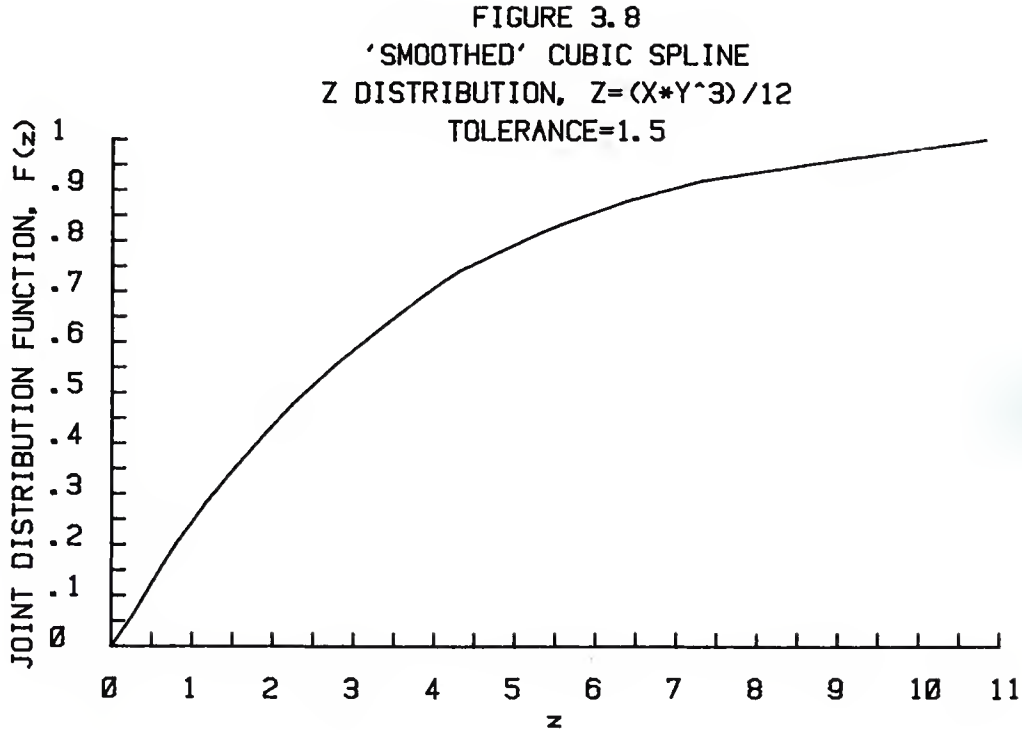


FIGURE 3.8
'SMOOTHED' CUBIC SPLINE
Z DISTRIBUTION, Z=(X*Y^3)/12
TOLERANCE=1.5
JOINT DISTRIBUTION FUNCTION, F(z)
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
0 1 2 3 4 5 6 7 8 9 10 11
z

The number of cells for the histogram is found using equation 3.9.

$$k = 1 + 3.3 * \log_{10} 25$$

$$k = 6$$

Using equation 3.10, the range is equal to

$$r = 10.8349 - 0.2587$$

$$r = 10.5762$$

And each cell has a width of

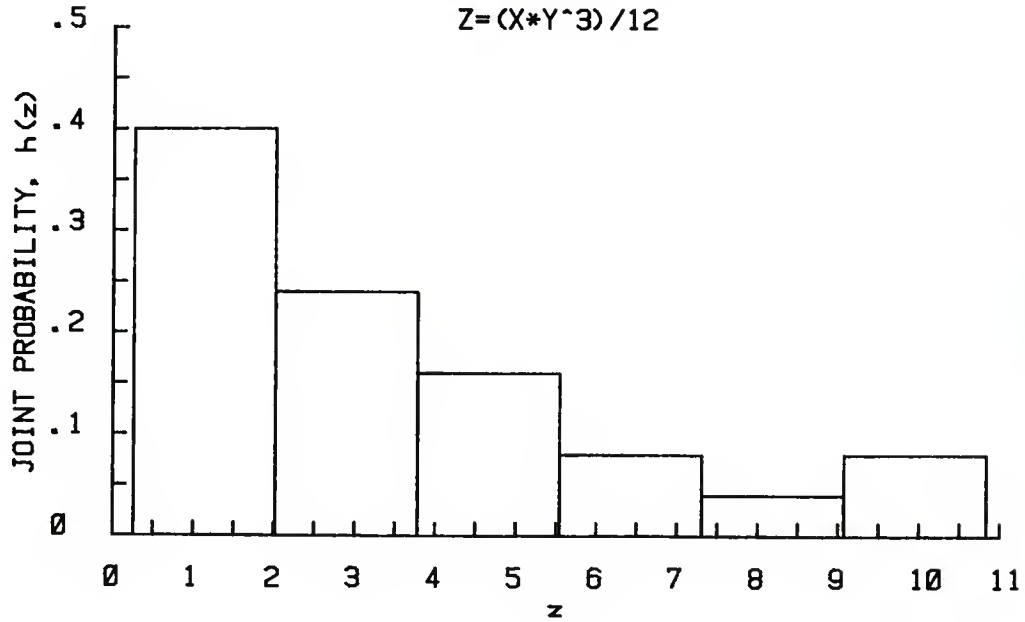
$$w = 10.5762/6$$

$$w = 1.7627$$

Figure 3.9 shows the histogram approximation for the function. The cubic spline density approximations are plotted in Figures 3.10 and 3.11. The cubic spline approximations are rough. However, if the number of cells per density function were increased, the cubic spline curves would tend to be smoother with less wiggle

It should be noted that this is a simplified case. In practice there could be an unlimited number of random variables with each random variable having 30 to 200 cells, resulting in literally thousands of calculations.

FIGURE 3.9
JOINT PROBABILITY
HISTOGRAM
Z=(X*Y^3)/12



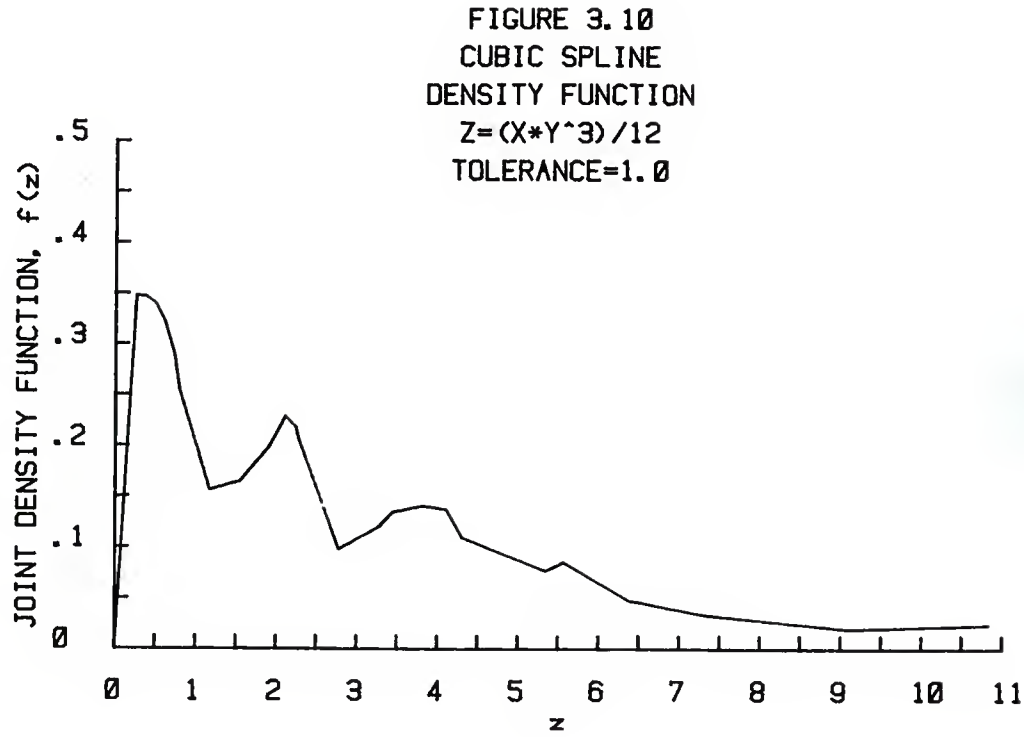
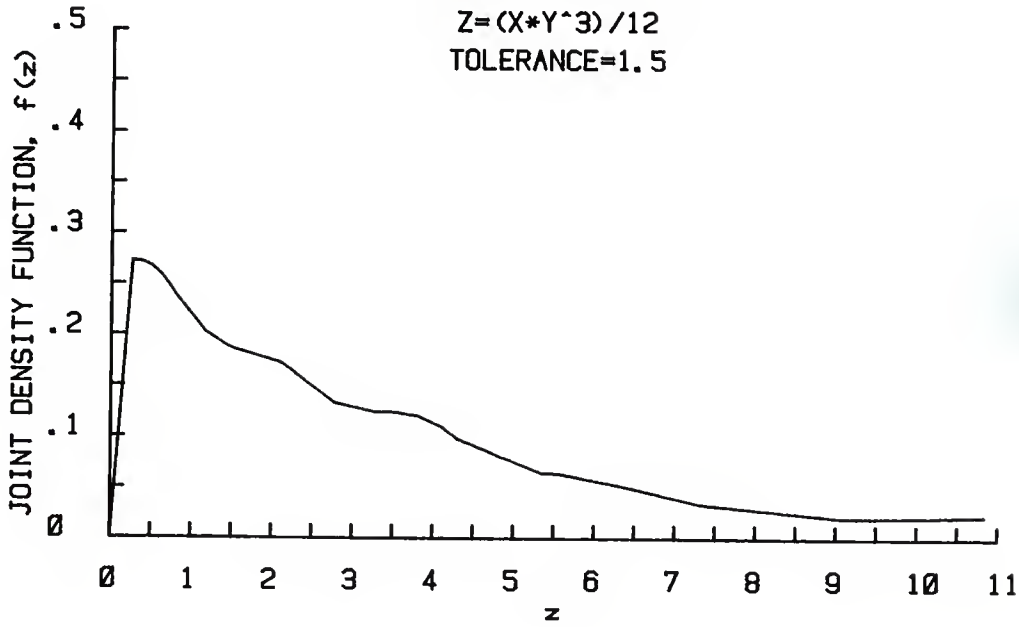


FIGURE 3.10
CUBIC SPLINE
DENSITY FUNCTION
Z=(X*Y^3)/12
TOLERANCE=1.0
JOINT DENSITY FUNCTION, f(z)
.5
.4
.3
.2
.1
0
0
1
2
3
4
5
6
7
8
9
10
11
z

FIGURE 3.11
CUBIC SPLINE
DENSITY FUNCTION
Z=(X*Y^3)/12
TOLERANCE=1.5



CHAPTER IV

PROGRAM DESCRIPTION

In this chapter, the computer programs that approximate the joint probability distribution and density functions are discussed. These programs are written in Fortran 77 on a Harris H-800 Super Mini-Computer. Because of storage problems, the simulation package had to be broken down into four separate programs. These programs are:

1. Discrete Simulation
2. Density Function Approximation
3. Data Management
4. Graphics

A complete listing of all the programs can be found in the Appendix.

DISCRETE SIMULATION

Program Name: DISCSIM

The discrete simulation program approximates the joint probability distribution function of two random variables with known probability density functions. Before executing the program, the desired function must be inserted in the "Function" subroutine. When run, the program prompts the user for the type of distribution of each random variable and the number of discrete points used to approximate each random variable's density function. The density functions available to the user are:

1. Normal: The normal distribution can be used to represent quantities such as measured errors.
2. Triangular: The triangular distribution is used in the absence of data to approximate a rough model of a random variable's density function.
3. Uniform: The uniform distribution is usually used as a first model for a variable that is suspected of being random.

4. Special: The special distribution is used if the user has a distribution function stored on file. e.g. If a distribution function was generated from experimental or measured results.

After the program has approximated the joint probability distribution function, the mean, variance, locations of the discrete points for each of the two random variables, and the joint probability distribution function of the algebraic expression is printed to work file "W2". The joint probability distribution is also printed to a file (designated by the user) that is compatible with the density program, the special distribution subroutine, the data management program, and the plotting programs.

Although this program simulates only two random variables, a joint probability distribution function of an algebraic expression containing more than two random variables can be approximated. To do this, the function must be separated into groups of two random variables. Using the special distribution subroutine, the joint probability distribution function of each group of two random variables can be combined to form the joint probability distribution function of the algebraic

expression. If an expression contains n random variables, the program must be run n-1 times to simulate the joint probability distribution function of the algebraic expression.

DENSITY FUNCTION APPROXIMATION

Program Name: DENSITY

The density function program is a compatible program used to approximate the density function after the joint probability distribution function has been simulated using the discrete simulation program. The density function is approximated using the first derivative of a cubic spline curve fit and a histogram (See Chapter 3). The density program prompts the user for the name of the joint distribution file. The user is able to control the tolerance of the cubic spline curve fit. With careful adjustment of the tolerance, a smooth density curve can be approximated. The histogram data is printed to work file "W1" and the spline data is printed to a file (designated by the user) that is compatible with the plotting programs.

DATA MANAGEMENT

Program Name: MANAGE

The data management program is used if a problem occurs with the calculation of the cubic spline approximation program. One of the drawbacks of the cubic spline curve fit is that if the data is grouped too close together, the program will halt because of a floating point overflow. The data management program sorts through the data and eliminates any data points that are closer than 0.000001 along the x axis.

GRAPHICS

Program Names: GRAPH

The plotting program is written using Precision Visuals' DI3000 computer graphics package. Program GRAPH plots the density and distribution functions on a Selanar Hirez 100 graphics terminal. The user is asked for the input data file name, title of the graph, and the X and Y axis labels. The plot is dumped to an Hewlett Packard 7470A plotter.

CHAPTER V

COMPARISON

In this chapter, the discrete simulation method discussed in Chapters 3 and 4 is compared to the Monte Carlo method. The joint distribution function of an equation is simulated using both methods and the results are compared to the exact solution of the joint distribution function. The accuracy of each method is measured using two parameters: Mean squared error and maximum deviation.

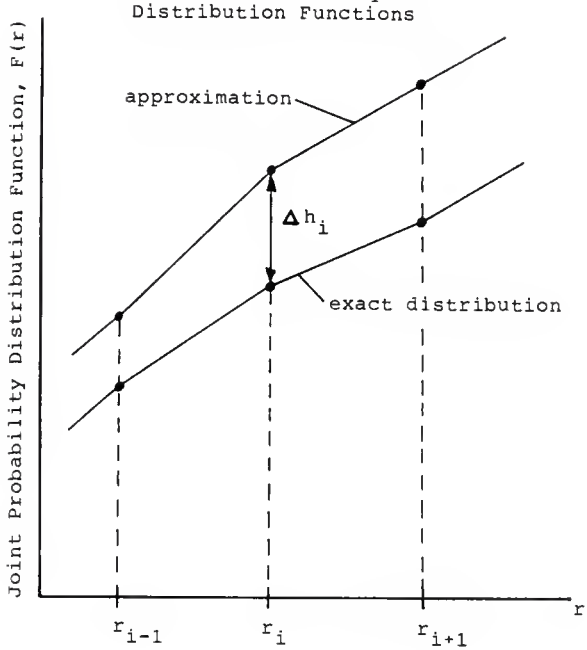
The mean squared error (MSE) is (See Figure 5.1),

$$MSE = \sum_{i=1}^{n} \frac{\Delta h_i^2}{n}$$

where n is the number of r locations. The maximum deviation is equal to the largest absolute value of Δh.

Figure 5.1

Approximation and Exact
Joint Probability
Distribution Functions



PROBLEM

Consider the equation of a circle,

$$r = \sqrt{X^2 + Y^2} \qquad 0 \leq r \leq \infty$$

where variables X and Y are normally distributed independent random variables. Each variable has a mean of zero and a standard deviation equal to one. Find the joint probability distribution function.

EXACT SOLUTION

The marginal probability density functions of X and Y are,

$$f_X(x) = \begin{cases} \frac{1}{2\pi\sigma^2} \exp\left[\frac{-(x-\mu)^2}{2\sigma^2}\right] & \text{for } -\infty \leq x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{2\pi\sigma^2} \exp\left[\frac{-(y-\mu)^2}{2\sigma^2}\right] & \text{for } \quad \leq y \leq \\ 0 & \text{otherwise} \end{cases}$$

When $\mu_X = 0$, $\sigma_X^2 = 1$ and $\mu_Y = 0$, $\sigma_Y^2 = 1$, the marginal

probability density functions become,

$$f_X(x) = \begin{cases} \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-x^2}{2}\right] & \text{for } -\infty \leq x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-y^2}{2}\right] & \text{for } -\infty \leq y \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

Since X and Y are mutually exclusive independent random variables, their joint probability density function is equal to the product of the two marginal probability density functions. Thus,

$$f(x,y) = f_X(x) f_Y(y)$$

So,

$$f(x,y) = \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-x^2}{2}\right] \frac{1}{\sqrt{2\pi}} \exp\left[\frac{-y^2}{2}\right]$$

$$= \frac{1}{2\pi} \exp\left[\frac{-(x^2+y^2)}{2}\right]$$

and the joint probability distribution is,

$$F(x,y) = \iint_D f(x,y)\,dxdy$$

$$= \frac{1}{2\pi} \int_0^r \int_0^{\sqrt{r^2-y^2}} \exp\left[-\frac{(x^2+y^2)}{2}\right] dxdy$$

in polar form this can be written as,

$$F_R(r) = P(R \leq r) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^r \exp\left[-\frac{r^2}{2}\right] r\,dr\,d\theta$$

to integrate by parts let,

$$u = r^2/2 \quad \text{and} \quad du = rdr$$

$$P(R \leq r) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^u \exp(-u)\,du\,d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} -\exp(-u) \Big|_0^u \, d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left[-\exp(-r^2/2)+1\right] d\theta$$

$$= \frac{1}{2\pi} \left[-\exp(-r^2/2)+1 \right] \theta \Big|_0^{2\pi}$$

$$\Pr(R \leq r) = 1 - \exp(-r^2/2)$$

The joint probability distribution function is,

$$F_R(r) = \begin{cases} 0 & r \leq 0 \\ 1 - \exp(-r^2/2) & 0 < r \leq \infty \end{cases}$$

The exact joint probability distribution function is plotted in Figure 5.2

Differentiating the joint probability distribution function yields the joint probability density function.

Figure 5.2
Exact Joint Probability
Distribution Function

$$f_R(r) = \begin{cases} 0 & r \leq 0 \\ r\exp(-r^2/2) & 0 < r \leq \infty \end{cases}$$

Figure 5.3 shows a plot of the exact joint probability density function.

MONTE CARLO SIMULATION vs. DISCRETE SIMULATION

A Monte Carlo computer program was developed to compare the accuracies of the two simulations with the exact solution. The Monte Carlo program generates random numbers that are normally distributed to approximate the X and Y random variable's density function. The subroutines used for generating the joint cumulative distribution function are the same subroutines used in the discrete approximation program.

The programs were run three times. The discrete simulation was run first. Each time an increasing number of discrete points was used to represent each random variable's density function. After each run, the number of unique r locations was noted. The Monte Carlo program was then run using the same number of unique r locations. A detailed explanation of the Monte Carlo method is located in Appendix G.

On the first run, 50 discrete points were used to approximate each random variable's density function.

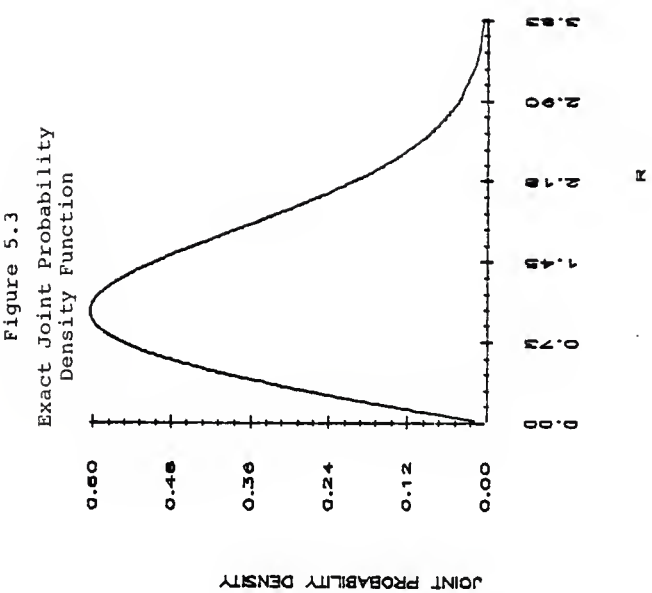


Figure 5.3
Exact Joint Probability
Density Function
JOINT PROBABILITY DENSITY
0.60
0.48
0.36
0.24
0.12
0.00
0.0
0.7
1.4
2.1
2.8
3.5
R

After all possible combinations were calculated, there were 325 unique r locations. Figure 5.4 shows the approximated joint probability distribution function of the discrete approximation. In Figure 5.5, the distribution function is plotted along with the exact solution. The first half of the discrete approximation is almost identical to the exact distribution function. However, in the upper 20% of the curve, the discrete approximation deviates quite a bit from the exact solution. The cause of the deviation could be the small number of discrete points used to represent the random variables density function. Because a relatively small number of points were used, the tails of the density functions were not adequately represented. The mean squared error and the maximum deviation for this approximation was 0.000028 and 0.0196446 respectively.

The Monte Carlo Simulation was then run using 325 unique r locations. The joint distribution function is plotted in Figure 5.6. Figure 5.7 shows the joint probability distribution function for the Monte Carlo approximation and the exact solution. As can be seen, the Monte Carlo approximation is not as good as the discrete approximation. The mean squared error is 0.0014709 and the maximum deviation is 0.0739843.

Figure 5.4

Joint Probability Distribution Function
for the Discrete Simulation using
50 Discrete Points per Random Variable

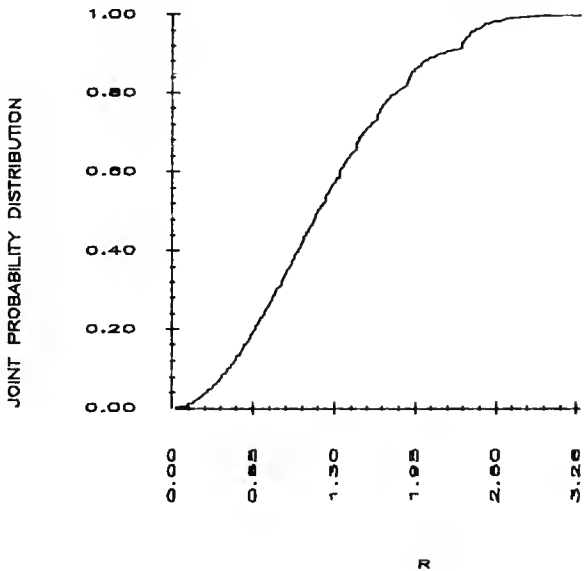


Figure 5.5

Joint Probability Distribution Functions
for the Discrete Simulation using
50 Discrete Points per Random Variable
and the Exact Solution

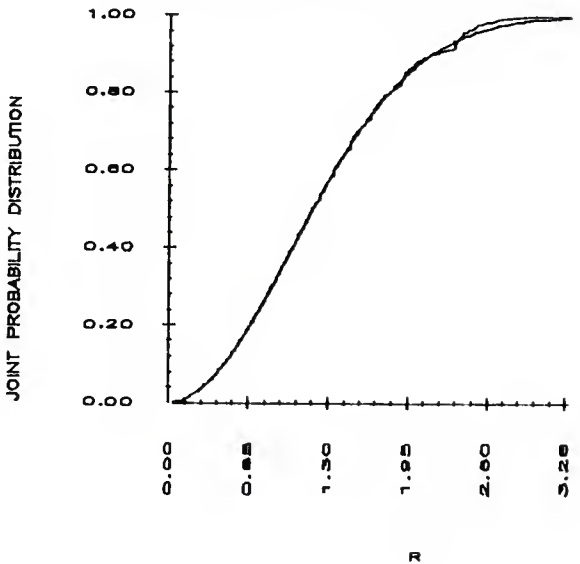


Figure 5.6

Joint Probability Distribution Function
for the Monte Carlo Simulation using
325 Unique R Locations

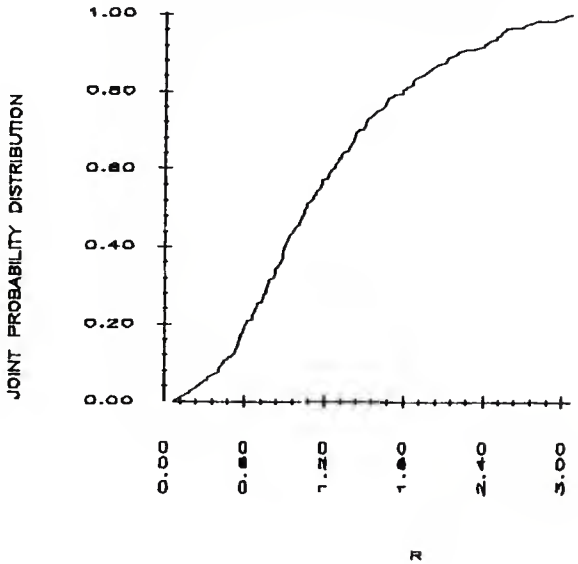
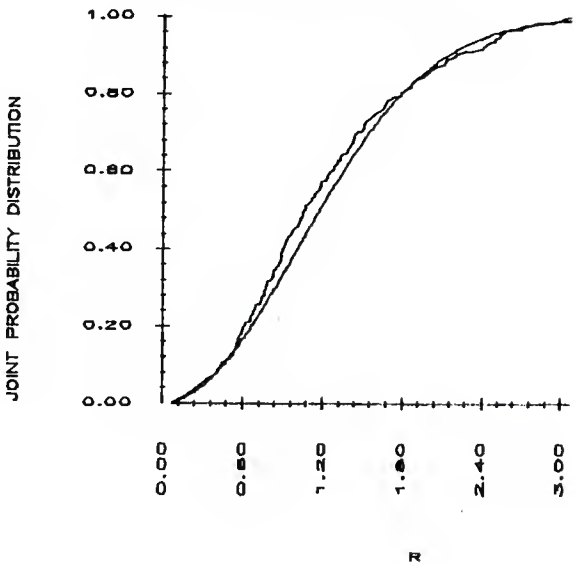


Figure 5.7

Joint Probability Distribution Functions
for the Monte Carlo Simulation using
325 Unique R Locations
and the Exact Solution



On the second run, 100 discrete points were used to approximate the X and Y random variable's density functions. This simulation resulted in 1275 unique r locations. Figure 5.8 shows the approximation of the second discrete approximation. In Figure 5.9, the discrete distribution curve is plotted with the exact solution. The curve still has a small dip at the upper portion of the curve. However, the maximum deviation has decreased to 0.0101403 and the mean squared error is 0.0000041. The 100 point approximation is an improvement over the 50 point approximation.

Figure 5.10 shows the Monte Carlo approximation of the joint distribution function for 1275 unique r locations. The distribution curve is plotted with the exact solution in Figure 5.11. Although the accuracy of the Monte Carlo simulation has improved, the approximation is still not as good as the discrete approximation. The maximum deviation is equal to 0.0314508 and the mean squared error is 0.0002180.

On the final run, 200 discrete points were used to approximate each variable's density function. This resulted in 5049 unique r locations. The joint distribution function is plotted in Figure 5.12. Figure 5.13 shows the approximated joint distribution function with the exact solution. The two distribution functions

Figure 5.8

Joint Probability Distribution Function
for the Discrete Simulation using
100 Discrete Points per Random Variable

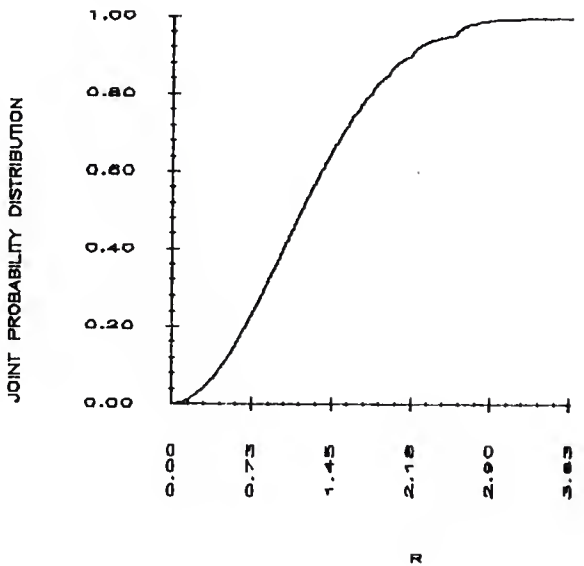


Figure 5.9

Joint Probability Distribution Functions
for the Discrete Simulation using
100 Discrete Points per Random Variable
and the Exact Solution

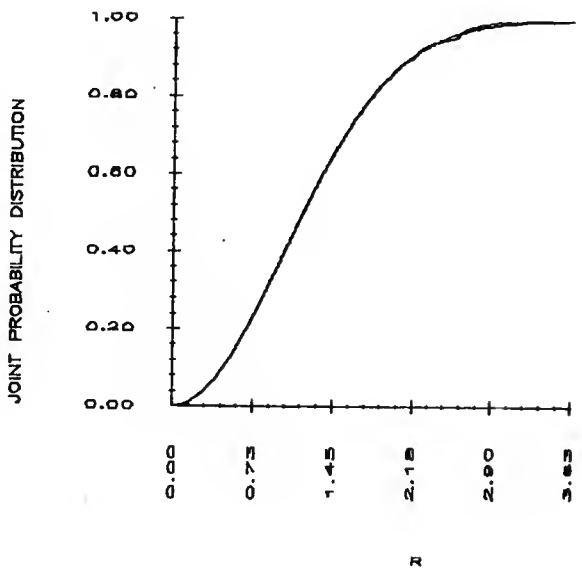


Figure 5.10

Joint Probability Distribution Function
for the Monte Carlo Simulation using
1275 Unique R Locations

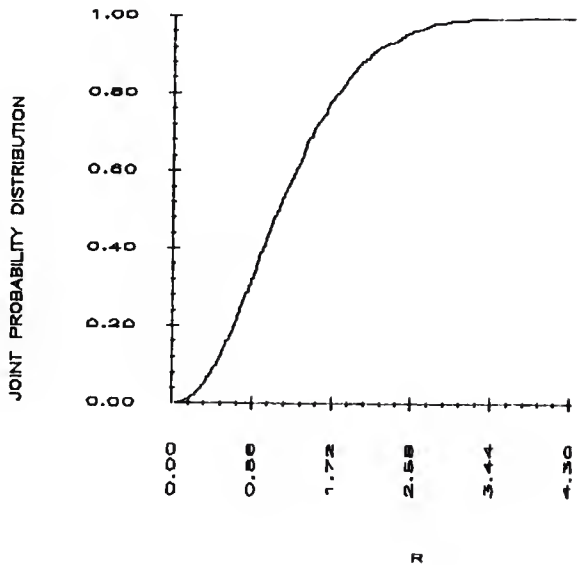


Figure 5.11

Joint Probability Distribution Functions
for the Monte Carlo Simulation using
1275 Unique R Locations
and the Exact Solution

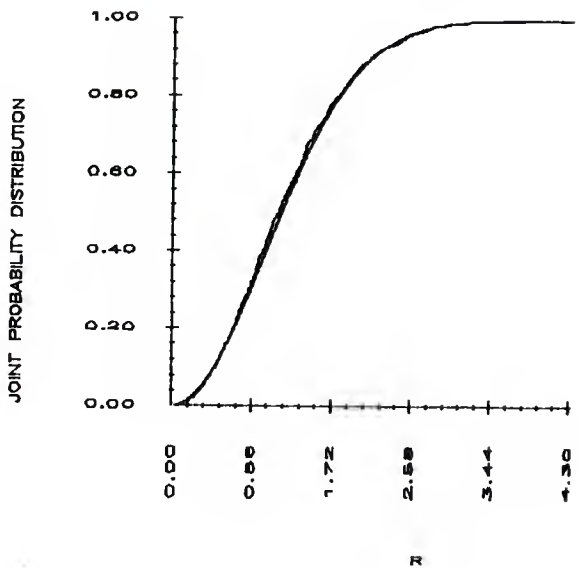


Figure 5.12

Joint Probability Distribution Function
for the Discrete Simulation using
200 Discrete Points per Random Variable

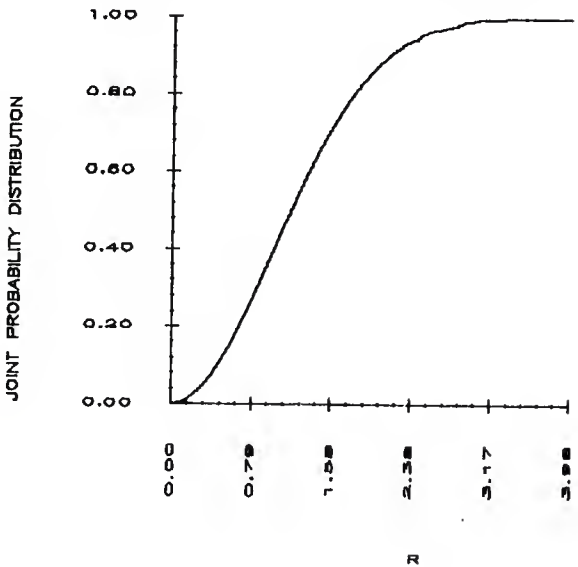
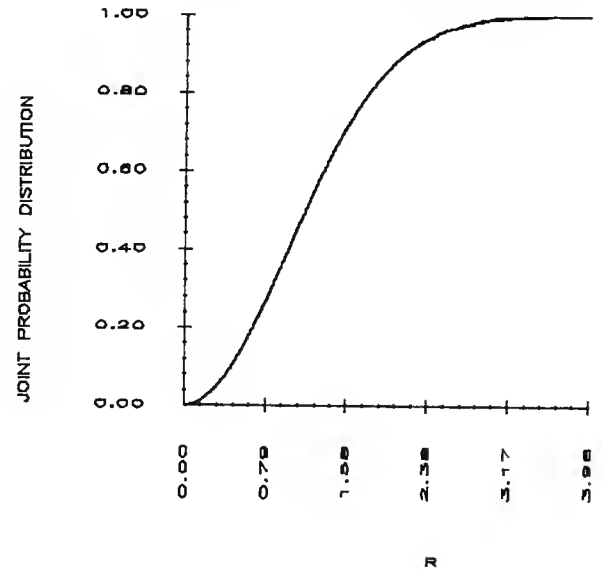


Figure 5.13

Joint Probability Distribution Functions
for the Discrete Simulation using
200 Discrete Points per Random Variable
and the Exact Solution



appear to be virtually identical. The dip that was present in the 50 and 100 point approximation is no longer noticeable. The error has been reduced to 0.0000006 and the maximum deviation is 0.0051283.

The Monte Carlo approximation for 5049 unique r locations is plotted in Figures 5.14 and 5.15. The Monte Carlo approximation has improved with an increase in r locations. Nevertheless, the discrete approximation is still more accurate than the Monte Carlo approximation. The mean squared error is 0.0000302, which is fifty times larger than the discrete approximation's error. The maximum deviation is three times larger than the discrete approximation at 0.0151759.

In Figure 5.16 and 5.17 the mean squared error and the maximum standard deviation is plotted as a function of unique r locations for both the discrete simulation program and the Monte Carlo program. In Figure 5.16, the discrete approximation approaches zero with little variation as the number of unique r locations increases. The Monte Carlo approximation decreased to zero also, however the distribution is scattered and unpredictable. In Figure 5.17, the maximum deviation of the two simulations decrease as the number of unique r locations increases. Again, the Monte Carlo data is scattered from one value to the next. The general trend of the Monte

Figure 5.14

Joint Probability Distribution Function
for the Monte Carlo Simulation using
5049 Unique R Locations

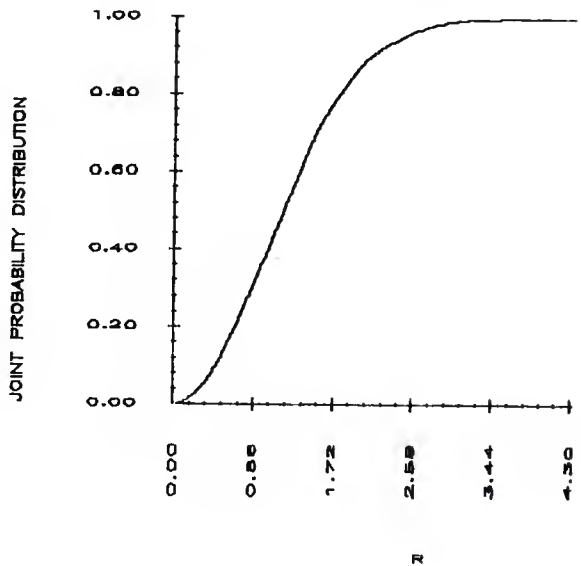


Figure 5.15

Joint Probability Distribution Functions for the Monte Carlo Simulation using 5049 Unique R Locations and the Exact Solution

Figure 5.16

Mean Squared Error vs.
Number of Unique R Locations
for the Monte Carlo and Discrete Simulation

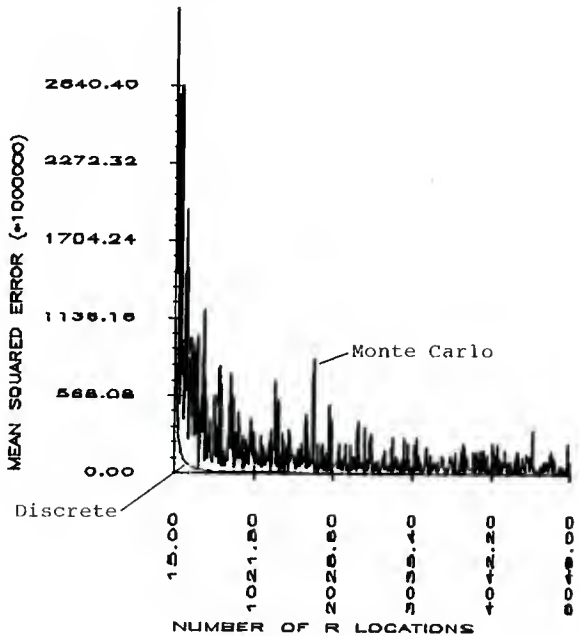
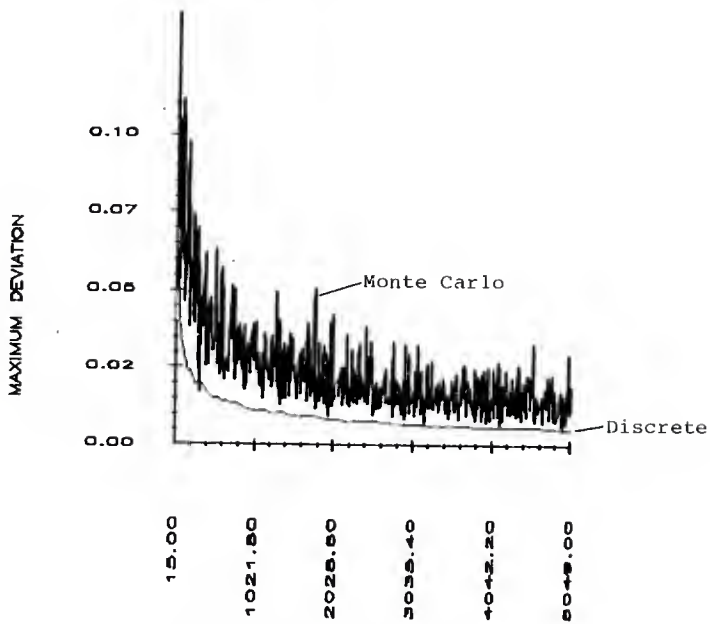


Figure 5.17

Maximum Deviation vs.
Number of Unique R Locations
for the Monte Carlo and Discrete Simulation



Carlo simulation is a decrease in the mean squared error and the maximum deviation as the number of r locations increase. The large scatter of error and deviation of the Monte Carlo method is caused by the statistical error associated with the generation of the random numbers. Because the generated numbers are random, the variation from one r location to the next is unpredictable.

In Figures 5.18 and 5.19 the mean squared error and the maximum deviation is plotted again for both the discrete simulation and the maximum deviation. However, in Figures 5.18 and 5.19 these values are plotted as functions of computer processing unit time. The figures are very similar to the previous two plots. The discrete simulation data is constantly decreasing and very predictable, while the Monte Carlo data is quite scattered.

A way to minimize the Monte Carlo scatter is to use a larger number of r locations. Increasing the number of r locations would lead to an increase of computing time and money. Therefor, the main advantage of the discrete simulation over the Monte Carlo simulation is that a predictable and more accurate representation of the joint probability distribution function can be approximated using fewer calculations and less computing time.

Figure 5.18

Mean Squared Error vs.
Computer Processing Unit Time
for the Monte Carlo and Discrete Simulation

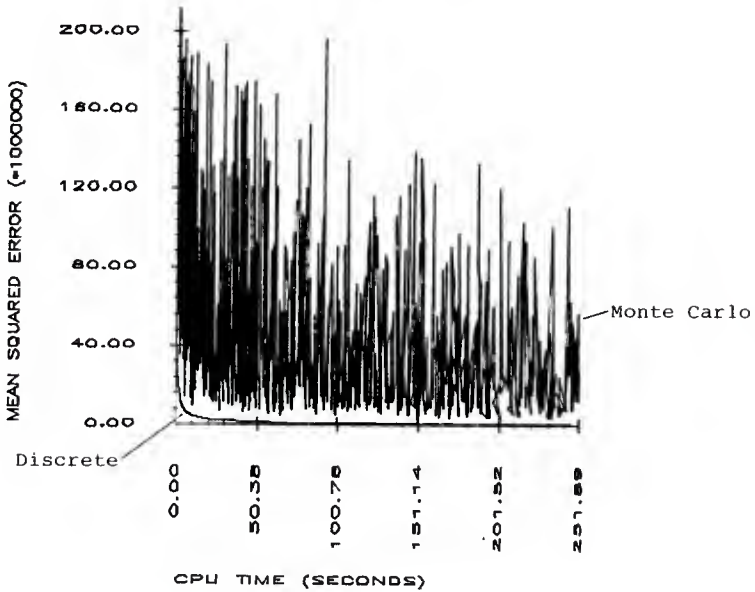
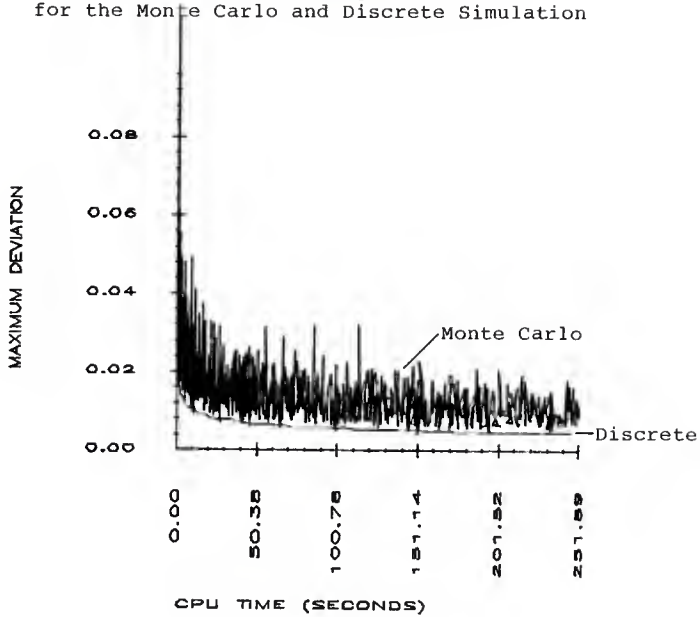


Figure 5.19

Maximum Deviation vs.
Computer Processing Unit Time
for the Monte Carlo and Discrete Simulation



CHAPTER VI

CANTILEVER I-BEAM

The beam problem discussed in this section was chosen to illustrate how the discrete simulation program could be used in an engineering application. Each random variable's density function was simulated using 100 discrete points. The "special" distribution subroutine was used to construct the joint probability density functions of algebraic expressions containing more than two random variables.

PROBLEM

The cantilever I-beam, shown in Figures 6.1 and 6.2, is made by welding three steel (ASTM A7) plates together. Determine:

a) The joint probability density and distribution functions for:

1. The maximum moment M
2. The deflection at B
3. The slope at B

Figure 6.1a

Cantilever I-Beam
Subjected to a Uniform Load, W_o

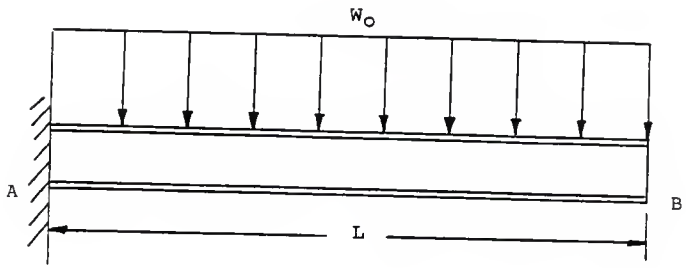


Figure 6.1b

Free-Body Diagram
of a Cantilever I-Beam
Subjected to a Uniform Load, W_o

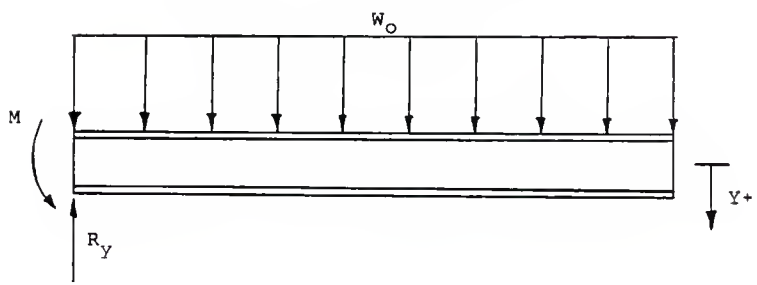
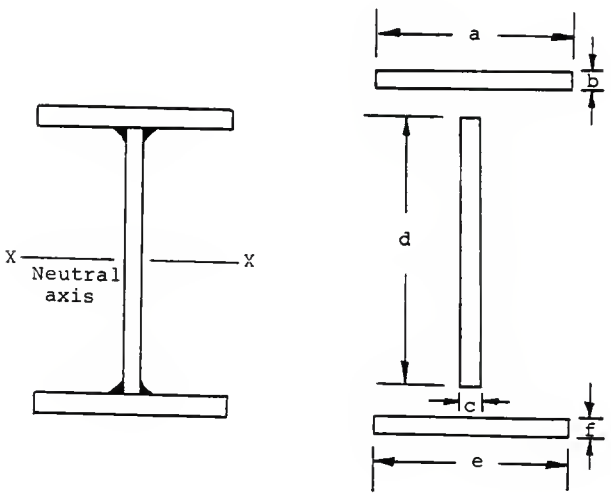


Figure 6.2
Cross Section of the Cantilever I-Beam



4. The maximum stress

5. The maximum shear

b) The reliability of the I-beam.

The dimensions and metal properties of the I-beam are assumed to be random variables with known probability density functions. These values are listed in Table 6.1.

Table 6.1
Random Variables for a Cantilever I-Beam

Random Variable	Mean	Standard Deviation	Units	Distribution
a	6.00	0.2500	inches	Normal
b	0.50	0.0625	inches	Normal
c	0.50	0.0625	inches	Normal
d	8.00	0.2500	inches	Normal
e	6.00	0.2500	inches	Normal
f	0.50	0.0625	inches	Normal
E	30E6	4.5E5	psi	Normal
L	16.00	1.0000	feet	Normal
W_o	45.00	33.3333	lb/inch	Uniform
S_u	65.6E3	2.43E3	psi	Normal
S_y	42.7E3	4.86E3	psi	Normal

SOLUTION

a) 1. The maximum moment for a cantilever beam with a uniform load can be expressed as,

$$M_{max} = \frac{W_o L^2}{2} \qquad (6.1)$$

Figures 6.3 and 6.4 show the joint probability distribution and density functions of the maximum moment. The maximum moment ranges from 59,877.02 to 78,638.19 foot-pounds with a mean value of 69,121.85 foot-pounds.

2. The maximum deflection at point B is equal to

$$Y_{max} = \frac{W_o L^4}{8EI} \qquad (6.2)$$

Before the deflection can be calculated, the joint probability distribution function of the moment of inertia must be approximated. Using the Parallel-Axis Theorem, the moment of inertia can be expressed as the equation,

$$I = (ab^3)/12 + ab(d/2 + b/2)^2 + (cd^3)/12 + (ef^3)/12 + fe(f/2 + d/2)^2 \qquad (6.3)$$

Figure 6.3
Joint Probability Distribution Function
of the Maximum Moment
of a Cantilever I-Beam

Figure 6.4
Joint Probability Density Function
of the Maximum Moment
of a Cantilever I-Beam

Figures 6.5 and 6.6 show the probability distribution and density functions of the moment of inertia. Once the moment of inertia's distribution has been determined, the joint probability density and distribution functions for the maximum deflection are approximated. Figures 6.7 and 6.8 show the joint probability distribution and density functions of the deflection at point B. The expected value of the deflection is 1.977 inches with a minimum possible value of 1.4 and a maximum value of 2.82 inches.

3. The slope ϕ, at point B can be written as,

$$\phi = \frac{W_o L^3}{6EI} \qquad (6.4)$$

The joint probability distribution and density functions are plotted in Figures 6.9 and 6.10. The discrete simulation predicts that the mean value of the slope at point B is 0.786 degrees, and the range of possible slope values is from 0.56 to 1.11 degrees.

4. The maximum stress of the cantilever I-beam occurs at the furthest distance from the neutral axis. The equation for the maximum stress is

$$S_{max} = \frac{M(d/2+f)}{I} \qquad (6.5)$$

Figure 6.5
Joint Probability Distribution Function
of the Moment of Inertia
of a Cantilever I-Beam

Figure 6.6

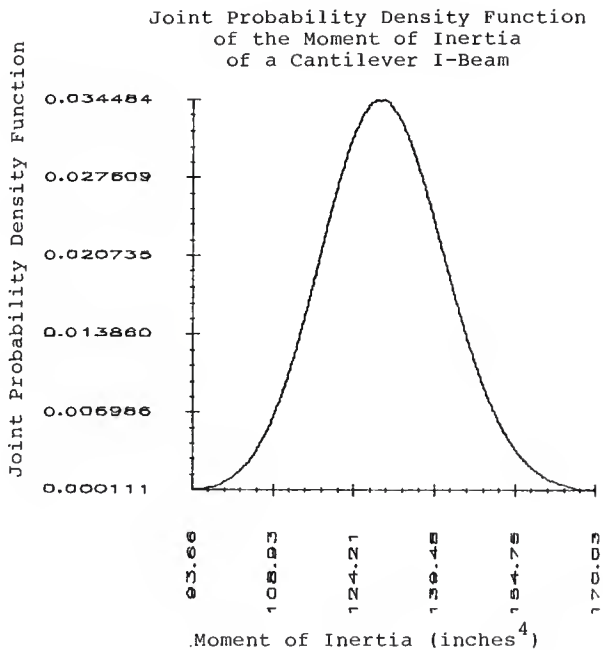

Joint Probability Density Function
of the Moment of Inertia
of a Cantilever I-Beam
Joint Probability Density Function
0.034484
0.027609
0.020735
0.013860
0.006986
0.000111
93.66
108.93
124.21
139.48
154.75
170.03
Moment of Inertia (inches4)

Figure 6.7

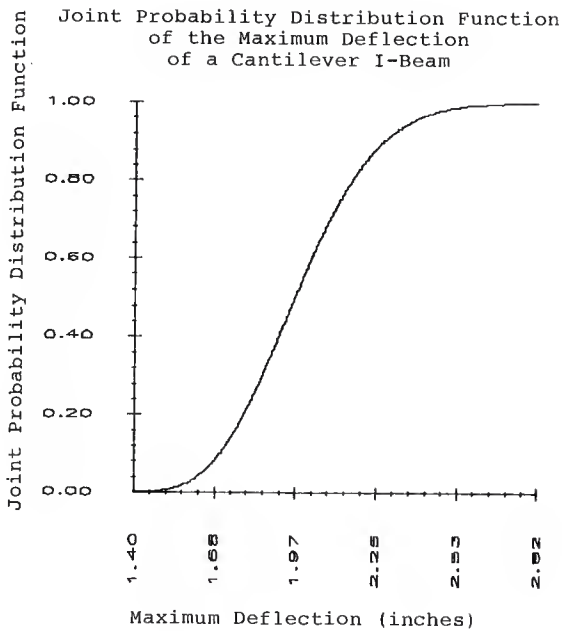

Joint Probability Distribution Function
of the Maximum Deflection
of a Cantilever I-Beam
Joint Probability Distribution Function
1.00
0.80
0.60
0.40
0.20
0.00
1.40
1.68
1.97
2.25
2.53
Maximum Deflection (inches)

Figure 6.8

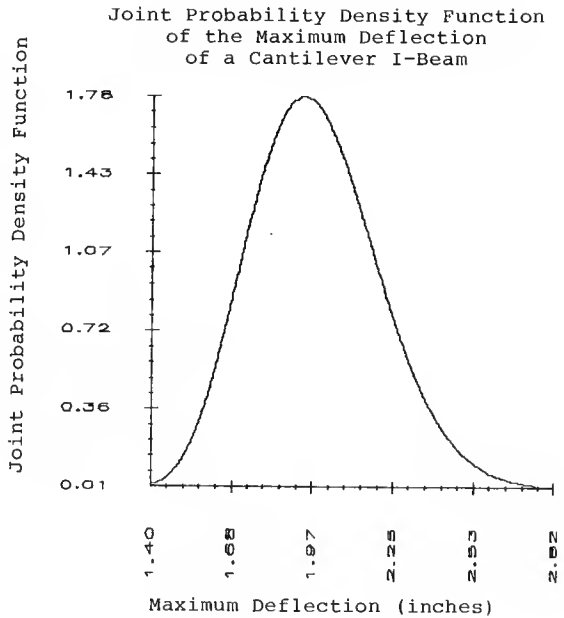

Joint Probability Density Function
of the Maximum Deflection
of a Cantilever I-Beam
Joint Probability Density Function
1.78
1.43
1.07
0.72
0.36
0.01
1.40
1.68
1.97
2.25
2.53
2.82
Maximum Deflection (inches)

Figure 6.9

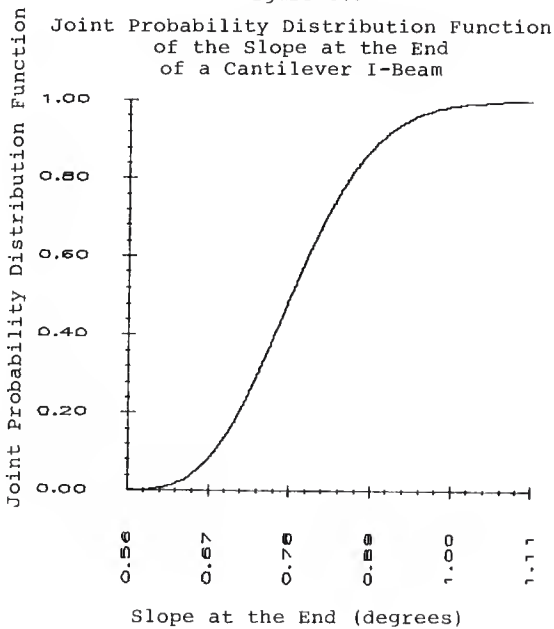
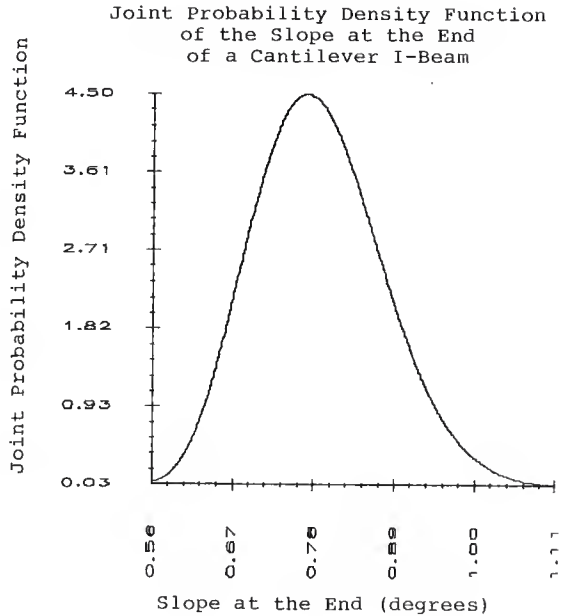

Joint Probability Distribution Function
of the Slope at the End
of a Cantilever I-Beam
Joint Probability Distribution Function
1.00
0.80
0.60
0.40
0.20
0.00
0.56
0.67
0.78
0.89
1.00
1.11
Slope at the End (degrees)

Figure 6.10


Joint Probability Density Function
of the Slope at the End
of a Cantilever I-Beam
Joint Probability Density Function
4.50
3.61
2.71
1.82
0.93
0.03
0.67
1.00
Slope at the End (degrees)

The expected value of the maximum stress is 28,944.26 psi. It is interesting to note that the range of possible maximum stress values differ by as much as 22,000 psi. The joint probability density and distribution functions are plotted in Figures 6.11 and 6.12.

5. The maximum value of the first moment of area occurs at the neutral axis. Since the width of the beam is thin at the neutral axis, the maximum shearing stress occurs at this point. The equation for the maximum shear stress can be written as

$$\tau_{max} = \frac{W_o L(ab(b/2+d/2) + cd^2/8)}{Ic} \qquad (6.6)$$

The expected value of the maximum shearing stress is 2,280.38 psi, and the range of possible shear values is from 1,171 to 4,441 psi. The joint probability density and distribution functions are plotted in Figures 6.13 and 6.14.

b) The reliability of the beam is equal to unity minus the probability of failure. If the stress in the beam exceeds the yield stress, the beam is presumed to have failed. Using this criteria, if the ratio of the maximum

Figure 6.11
Joint Probability Distribution Function
of the Maximum Stress
of a Cantilever I-Beam

Figure 6.12

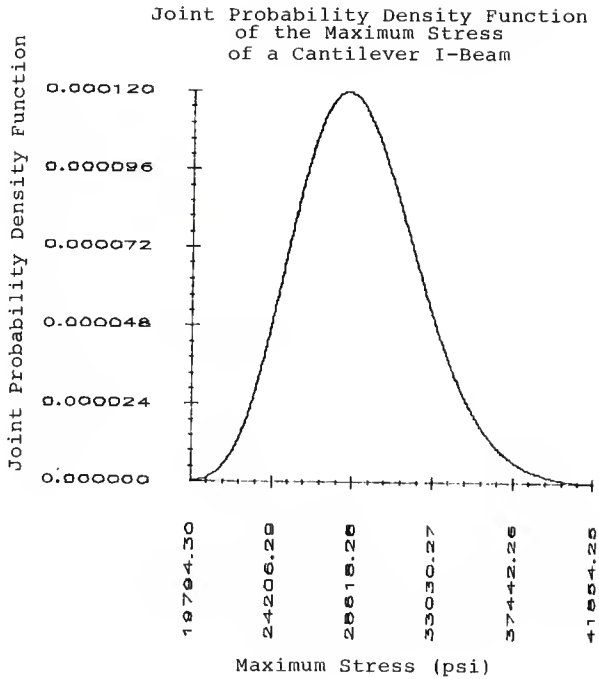
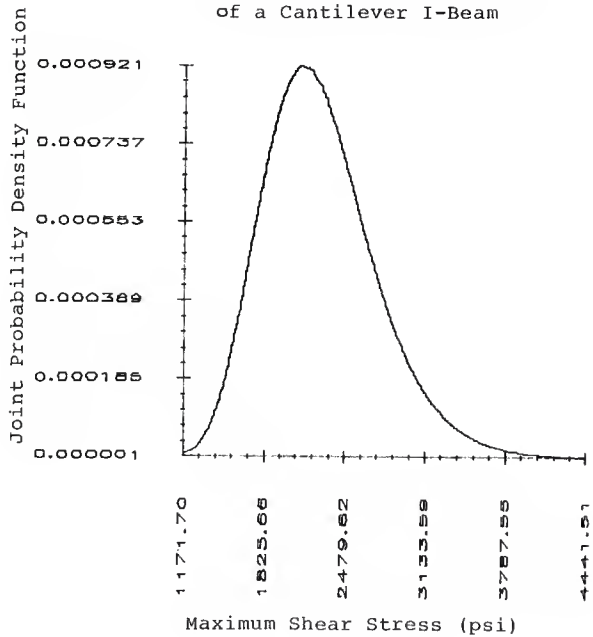

Joint Probability Density Function
of the Maximum Stress
of a Cantilever I-Beam
Joint Probability Density Function
0.000120
0.000096
0.000072
0.000048
0.000024
0.000000
19704.30
24206.29
28618.28
33030.27
37442.26
41854.25
Maximum Stress (psi)

Figure 6.13
Joint Probability Distribution Function
of the Maximum Shear Stress
of a Cantilever I-Beam

Figure 6.14

Joint Probability Density Function
of the Maximum Shear Stress
of a Cantilever I-Beam



stress in the beam to the yield stress is greater than one, failure occurs. The joint probability and density functions for the stress ratio are plotted in Figures 6.15 and 6.16. Using the joint probability distribution, the reliability of the system can be determined. From Figure 6.15 the probability of the stress ratio being less than or equal to one is equal to approximately 98%. Therefore, the probability of failure is roughly 2% and the probable reliability of the I-beam structure is 98%.

Figure 6.15

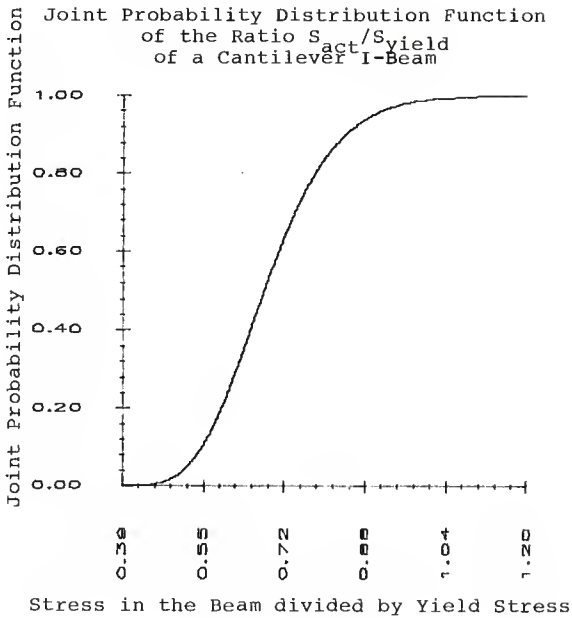
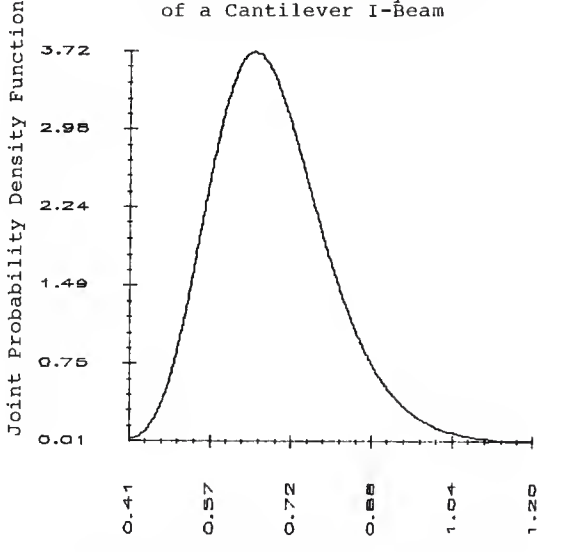

Joint Probability Distribution Function
of the Ratio S_{act}/S_{yield}
of a Cantilever I-Beam
Joint Probability Distribution Function
1.00
0.80
0.60
0.40
0.20
0.00
0.55
Stress in the Beam divided by Yield Stress

Figure 6.16

Joint Probability Density Function
of the Ratio S_{act}/S_{yield}
of a Cantilever I-Beam



CHAPTER VII

BOUNDING

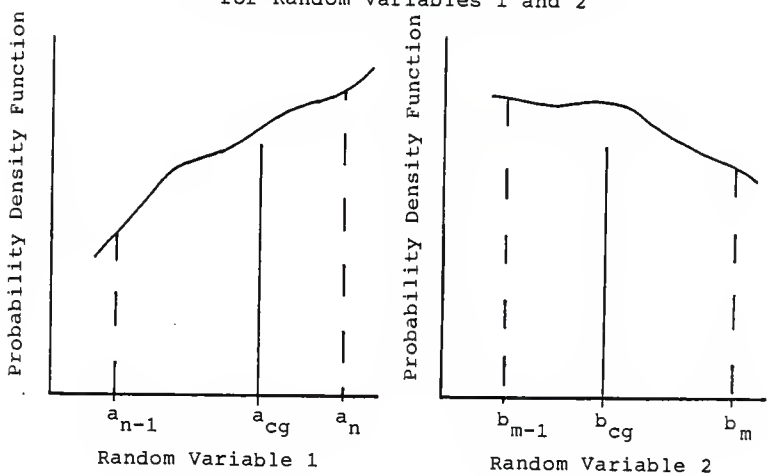
During the development of the discrete simulation algorithm, a new technique was investigated that approximates the upper and lower bounds of a joint probability distribution function of an algebraic expression containing several independent random variables. Although a formal proof of the bounding theory has not been written, the results of the program have been compared with several known solutions with good results. In this chapter, the procedure used to obtain the upper and lower bounds of a joint probability distribution function is discussed.

Using the same method of cell development as the discrete simulation technique discussed in Chapter 3, two curves are developed that bound the exact joint probability distribution function of an algebraic expression. The bounding approximation divides the two known random variables' density functions into a given

number of cells. The upper and lower bounds of each cell are noted as well as the cell's center of gravity (See Figure 7.1).

Figure 7.1

Known Probability Density Functions for Random Variables 1 and 2



M and n are arbitrary cell divisions within the random variables', X_1 and X_2, known density functions. Since each cell is represented using three locations,

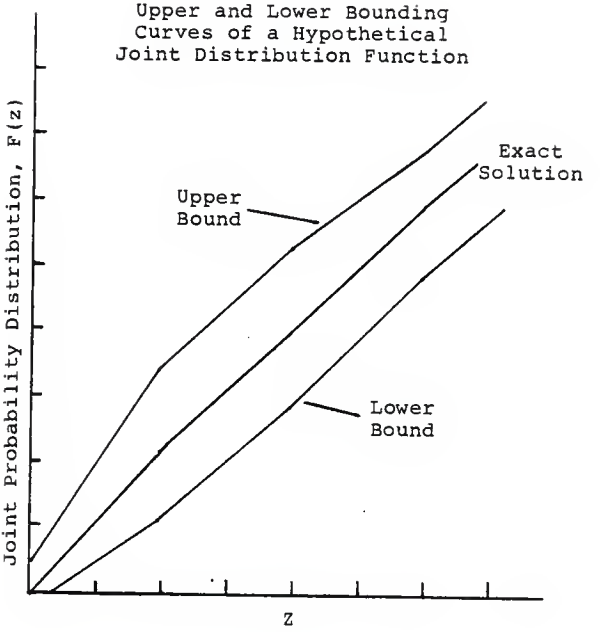
there are nine possible outcomes that need to be considered.

$$Z_1 = Z(X_1(a_{n-1}), X_2(b_{m-1})) \qquad (7.1)$$
$$Z_2 = Z(X_1(a_{n-1}), X_2(b_{cg}))$$
$$Z_3 = Z(X_1(a_{n-1}), X_2(b_m))$$
$$Z_4 = Z(X_1(a_{cg}), X_2(b_{m-1}))$$
$$Z_5 = Z(X_1(a_{cg}), X_2(b_{cg}))$$
$$Z_6 = Z(X_1(a_{cg}), X_2(b_m))$$
$$Z_7 = Z(X_1(a_n), X_2(b_{m-1}))$$
$$Z_8 = Z(X_1(a_n), X_2(b_{cg}))$$
$$Z_9 = Z(X_1(a_n), X_2(b_m))$$

The nine values are compared with each other. The maximum value is stored in the lower Z bounding array and the minimum value is stored in the upper Z bounding array. Each bounding distribution is then assembled independently using the same distribution and sort subroutines used for the discrete approximation. Figure 7.2 shows the upper and lower bounds of a hypothetical joint distribution function.

Figure 7.2

Upper and Lower Bounding Curves of a Hypothetical Joint Distribution Function



The bounding program was compared with the exact solution of the probabilistic pythagorean random variable discussed in Chapter 5. The bounding approximation bounded both the exact solution as well as the approximated joint probabilistic distribution function.

Figure 7.3 shows the exact solution and the upper and lower bounding curves using 100 cells per random variable.

The bounding routine was also compared to Wu's fast probability integration (FPI) approximation 23 of the probability of failure of a bar in tension. The probability of failure, P_f is symbolized as

$$P_f = P(g \leq 0) \qquad (7.2)$$

where g is equal to the equation

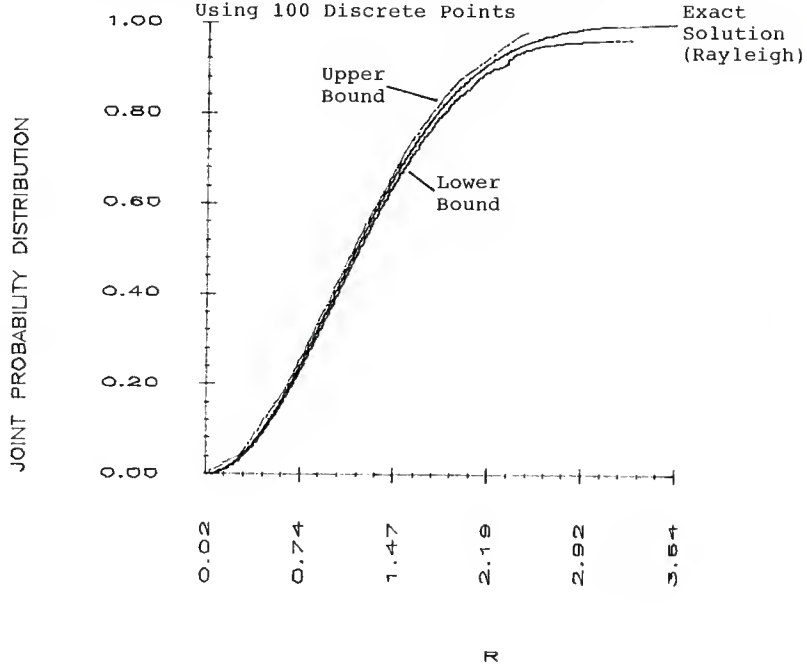
$$g = R - (4T/\pi D^2) \qquad (7.3)$$

R and D are independent normal random variables. R has a mean of 170.0 N/mm and a variance of 625.0 N/mm . D has a mean and variance of 29.4 mm and 9.0 mm respectively. T is equal to 50 kN. Table 7.1 lists Wu's approximation as well as the exact solution and the upper and lower bounding approximations. An example of the output using ten cells per random variable is listed in the Appendix.

Table 7.1 shows that as the number of cell divisions per density function increase for each random variable,

Figure 7.3

The Exact Solution and the Bounding Curves for the Probabilistic Pythagorean Random Variable Using 100 Discrete Points



the upper and lower bounds converge toward the exact solution.

Table 7.1

Bounding Probabilities of Failure of a Bar in Tension

Number of Discrete Points/ Density Function	Lower Bound	Exact Solution	FPI Approx-imation	Upper Bound
100	0.00030	0.00230	0.00229	0.0114
120	0.00032			0.0098
150	0.00043			0.0083
200	0.00061			0.0068

The subroutines, Bookbound and Outputbound, that approximate the upper and lower bounds are listed in the Appendix.

CHAPTER VIII

CONCLUSIONS

The purpose of this study was to develop a computer simulation package that is capable of approximating the joint probability distribution function of a complex algebraic expression containing several independent random variables. A discrete simulation technique was developed and implemented on a Harris H-800 Super Mini-Computer.

In Chapter 3 the discrete simulation technique is discussed. Although the discrete simulation method is similar to the Monte Carlo technique, the discrete method divides each random variable's known density function into a given number of n discrete sections with equal areas. Each area is then represented by a single discrete point with a probability of 1/n. Using the discrete points, all possible combinations of the algebraic expression are simulated. After all the outcomes have been computed, the joint distribution

function is assembled by sorting the data and summing the ordered values probabilities. The approximate shape of the density function is simulated using cubic splines and a histogram.

The computer programs that are used to approximate the joint probability distribution and density functions are described in Chapter 4. Because of storage problems, the simulation package had to be divided into four separate programs. The programs are listed in the Appendix.

In Chapter 5 the discrete approximation is compared to the traditional Monte Carlo technique for a joint probability distribution that can be solved exactly. The joint probability distribution function of the equation of a circle is approximated using both methods and the results are compared to the exact solution of the joint distribution function. The programs were run three times. Each time an increasing number of points was used to represent each random variable. The joint probability distributions are plotted vs. the exact solution for each case. As expected, the accuracy of each method improves with an increase of number of points used to represent each random variable's density function. However, the accuracy of the discrete simulation is better for each

case than the Monte Carlo simulation. The mean squared error and the maximum deviation is plotted as a function of number of unique outcomes and computer processing unit (CPU) time. The discrete simulation plots are very well behaved and consistently decreasing with an increasing number of outcomes and CPU time, while the Monte Carlo data is scattered and erratic.

In Chapter 6 an example of how the discrete simulation method can be used as a tool by engineers to aid in the design and development of structures and components is presented. A cantilever I-beam is analyzed. Each variable is assumed to be random with a known probability distribution. Using the discrete simulation programs, the joint probability and density functions are approximated for the maximum moment, the deflection at the end of the beam, the slope at the end of the beam, the maximum stress, the maximum shear, and the reliability of the beam.

In Chapter 7 a different technique is discussed that determines the upper and lower bounds of a joint probability distribution function using the discrete approximation. Although a formal proof has not been developed for the bounding theory, the program was run for several known cases with good results.

In summary, a discrete simulation algorithm was developed to approximate the joint probability distribution function of complex algebraic expressions containing several independent random variables. Although there are a few methods available for approximating the joint probability distribution functions of algebraic expressions, they usually involve complex mathematical operations that are too cumbersome for the average design engineer. The discrete simulation method developed in this study does not involve any complex mathematical operations, is easy to use, and is capable of being used in a wide variety of engineering applications.

CHAPTER IX

RECOMMENDATIONS FOR FURTHER STUDY

There are many possible extensions of the discrete simulation method that can be investigated following this study. A few suggestions are to:

1. Develop a faster sorting routine which would decrease computing time.
2. Expand the program to allow the use of more discrete points to represent each random variable's density function.
3. Modify the program so that the variables are represented using double precision.
4. Investigate the accuracy of the discrete simulation program for an algebraic expression containing more than two random variables in which an exact solution exists.
5. Develop a formal proof of the bounding theory discussed in Chapter 7.

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APPENDIX A

DISCRETE SIMULATION PROGRAM

```
C ************************************************************************
C *                                                                      *
C * PROGRAM NAME: DISCSIM (DISCRETE SIMULATION AND BOUNDING)             *
C *                                                                      *
C * WRITTEN BY: BRUCE SWANSON                                            *
C *                                                                      *
C * DATE: MARCH 26, 1987                                                 *
C *                                                                      *
C * PURPOSE: PROCRAM "DISCSIM" APPROXIMATES THE JOINT PROBABILITY        *
C *          DISTRIBUTION FUNCTION OR THE UPPER AND LOWER BOUNDING       *
C *          CURVES OF A DISTRIBUTION FUNCTION FOR AN ALGEBRAIC          *
C *          EXPRESSION CONTAININC TWO INDEPENDENT RANDOM VARIABLES.     *
C *          THE RANDOM VARIABLE'S DENSITY FUNCTIONS ARE APPROXIMATED    *
C *          USING A FINITE NUMBER (2-200) OF DISCRETE LOCATIONS.  THE   *
C *          FUNCTION MUST BE INSERTED IN SUBROUTINE "FUNCTION".         *
C *                                                                      *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINES BOOK               *
C *                                                   BOOKBOUND          *
C *                                                   CELLR              *
C *                                                   DIST               *
C *                                                   DISTRIBUTION       *
C *                                                   FUNCTION           *
C *                                                   NORMAL             *
C *                                                   NORMINV            *
C *                                                   OUTPUT             *
C *                                                   OUTPUTBOUND        *
C *                                                   SORT               *
C *                                                   SPECIAL            *
C *                                                   TRIANCLE           *
C *                                                   UNIFORM            *
C *                                                                      *
C * DESCRIPTION OF PARAMETERS:                                           *
C *     INPUT ARCUMENTS:                                                 *
C *             ANSB - CHARACTER*1 - IF ANSB IS EQUAL TO 'Y' THEN THE    *
C *                    PROGRAM COMPUTES THE UPPER AND LOWER BOUNDS OF    *
C *                    THE JOINT DISTRIBUTION FUNCTION AS WELL AS THE    *
C *                    DISCRETE APPROXIMATION FUNCTION.  IF ANSB IS      *
C *                    EQUAL TO 'N' THEN THE PROCRAM COMPUTES ONLY THE   *
C *                    DISCRETE APPROXIMATION OF THE JOINT               *
C *                    DISTRIBUTION FUNCTION.                            *
C *             DISREP(NUM) - CHARACTER*10 - DISREP CONTAINS THE         *
C *                    TYPE OF PROBABILITY DISTRIBUTION FOR VARIABLE     *
C *                    NUM (NORMAL, UNIFORM, TRIANGULAR, SPECIAL).       *
C *             DISTR(NUM) - INTEGER - THE NUMBER OF THE DISTRIBUTION    *
C *                    FOR RANDOM VARIABLE NUM (1=NORMAL, 2=UNIFORM,     *
C *                     3=TRIANCULAR, 4=SPECIAL).                        *
C *             NAME - CHARACTER*30 - NAME IS THE INPUT DATA FILE NAME   *
C *                    OF THE SPECIAL DISTRIBUTION FUNCTION FOR          *
```

```
C *                RANDOM VARIABLE NUM.                                  *
C *          NAME2 - CHARACTER*30 - NAME2 IS THE OUTPUT DATA FILE        *
C *                NAME OF THE JOINT DISTRIBUTION FUNCTION.              *
C *          NCELLS(NUM) - INTECER - THE NUMBER OF DISCRETE              *
C *                LOCATIONS THAT APPROXIMATE THE PROBABILTIY            *
C *                DENSITY FUNCTION OF RANDOM VARIABLE NUM.              *
C *          NUM - INTEGER - THE NUMBER OF THE RANDOM VARIABLE.          *
C *          VARIANCE - REAL - THE VARIANCE VALUE FOR RANDOM             *
C *                VARIABLE NUM.                                         *
C *                                                                      *
C *     OUTPUT ARGUMENTS:                                                *
C *          A(N) - REAL - WHERE N=NCELLS*2, ODD VALUES OF N             *
C *                REPRESENT THE CENTER OF GRAVITY LOCATIONS AND         *
C *                EVEN VALUES OF N INDICATE THE CELL BOUNDARIES.        *
C *          B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF        *
C *                CRAVITY) REPRESENTATIONS FOR NUM'S PROBABILITY        *
C *                DENSITY FUNCTION.                                     *
C *          COUNT - INTECER - COUNT IS THE NUMBER OF UNIQUE Z           *
C *                LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION       *
C *                FUNCTION.                                             *
C *          LB(NCELLS( ),2) - REAL - CONTAINS THE LOWER BOUND           *
C *                LOCATION FOR CELL NCELLS( ).                          *
C *          LBCOUNT - INTEGER - LBCOUNT IS THE NUMBER OF UNIQUE         *
C *                LB( , ) LOCATIONS OF THE LOWER BOUND                  *
C *                DISTRIBUTION FUNCTION.                                *
C *          MEAN(NUM) - REAL - THE MEAN VALUE FOR RANDOM VARIABLE       *
C *                NUM.                                                  *
C *          UB(NCELLS( ),2) - REAL - CONTAINS THE UPPER BOUND           *
C *                LOCATION FOR CELL NCELLS( ).                          *
C *          UBCOUNT - INTEGER - UBCOUNT IS THE NUMBER OF UNIQUE         *
C *                UB( , ) LOCATIONS OF THE UPPER BOUND                  *
C *                DISTRIBUTION FUNCTION.                                *
C *          Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT                *
C *                PROBABILITY DISTRIBUTION FUNCTION.                    *
C *          Z(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS         *
C *                THAN OR EQUAL TO Z(COUNT,1).                          *
C *          Z(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF        *
C *                TIMES Z(COUNT,1) WAS REPEATED DURINC THE              *
C *                SIMULATION.                                           *
C *          ZLB(COUNT,1) - REAL - THE Z VALUE OF THE LOWER BOUND        *
C *                OF THE JOINT PROBABILITY DISTRIBUTION FUNCTION.       *
C *          ZLB(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS       *
C *                THAN OR EQUAL TO ZLB(COUNT,1).                        *
C *          ZLB(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF      *
C *                TIMES ZLB(COUNT,1) WAS REPEATED DURING THE            *
C *                SIMULATION.                                           *
C *          ZMEAN - REAL - THE MEAN OF THE JOINT PROBABILITY            *
C *                DISTRIBUTION FUNCTION.                                *
```

```
C *            ZSTANDARD_DEV - REAL - THE STANDARD DEVIATION OF THE    *
C *                  JOINT PROBABILITY DISTRIBUTION Z.                 *
C *            ZUB(COUNT,1) - REAL - THE Z VALUE OF THE UPPER BOUND    *
C *                  OF THE JOINT PROBABILITY DISTRIBUTION FUNCTION. *
C *            ZUB(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS   *
C *                  THAN OR EQUAL TO ZUB(COUNT,1).                    *
C *            ZUB(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF  *
C *                  TIMES ZUB(COUNT,1) WAS REPEATED DURING THE        *
C *                  SIMULATION.                                       *
C *            ZVARIANCE - REAL - THE VARIANCE OF THE JOINT            *
C *                  PROBABILITY DISTRIBUTION FUNCTION.                *
C *                                                                    *
C **********************************************************************
C
      INTEGER COUNT,BX,DISTR,NCELLS,NUM,N,UBCOUNT,LBCOUNT
      REAL A,B,Z,AREA,PROB,X,TOLERANCE,F,D,SX,W
      REAL VARIANCE,MEAN,ZMEAN,ZVARIANCE,UB,LB
      CHARACTER DISREP*10,NAME*20,NAME2*20,ANSB*1
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      OPEN (UNIT=23,FILE='W2')
320   WRITE (3,*)
      WRITE (3,300)
300   FORMAT (2X,'SIMULATION AND BOUNDING (Y/N) ?')
      READ (0,310) ANSB
310   FORMAT (A1)
      IF (ANSB.NE.'Y'.AND.ANSB.NE.'N') THEN
         GOTO 320
      END IF
      DO 10 NUM=1,2
         CALL DIST
10    CONTINUE
      IF (ANSB.EQ.'Y') THEN
         CALL BOOKBOUND
         CALL OUTPUTBOUND
      ELSE
         CALL BOOK
         CALL OUTPUT
      END IF
      CLOSE 23
      STOP
      END
```

```
      SUBROUTINE BOOK
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE BOOK                                           *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 4, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE BOOK SIMULATES THE JOINT DISTRIBUTION FUNCTION      *
C *          OF TWO RANDOM VARIABLES FOR THE DISCRETE APPROXIMATION.        *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINES DISTRIBUTION          *
C *                                                   FUNCTION              *
C *                                                   SORT                  *
C *                                                                         *
C * USAGE: CALL BOOK                                                        *
C *                                                                         *
C * DISCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *           B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF          *
C *                  GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILITY         *
C *                  PROBABILITY DENSITY FUNCTION.                          *
C *           NCELLS(NUM) -  INTEGER - THE NUMBER OF DISCRETE               *
C *                  LOCATIONS USED TO APPROXIMATE THE PROBABILITY          *
C *                  DENSITY FUNCTION OF RANDOM VARIABLE NUM.               *
C *           NUM - INTEGER - VARIABLE NUM IS THE NUMBER OF THE             *
C *                  RANDOM VARIABLE.                                       *
C *     OUTPUT ARGUMENTS:                                                   *
C *           COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z             *
C *                  LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION        *
C *                  FUNCTION.                                              *
C *           TIME - INTEGER*3 - VARIABLE TIME IS THE TOTAL CPU TIME        *
C *                  OF SIMULATION.                                         *
C *           Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT                  *
C *                  PROBABILITY DISTRIBUTION FUNCTION.                     *
C *           Z(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS           *
C *                  THAN OR EQUAL TO Z(COUNT,1).                           *
C *           Z(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF          *
C *                  TIMES Z(COUNT,1) WAS REPEATED DURING THE               *
C *                  SIMULATION.                                            *
C *           ZMEAN - REAL - THE MEAN OF THE JOINT PROBABILTIY              *
C *                  DISTRIBUTION FUNCTION.                                 *
C *           ZVARIANCE - REAL - THE VARIANCE OF THE JOINT                  *
C *                  PROBABILITY DISTRIBUTION FUNCTION.                     *
C *                                                                         *
C ***************************************************************************
C
```

```
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      REAL B,Z,SX,F,D,WORK,A,VARIANCE,MEAN,ZMEAN,ZVARIANCE,TOTAL
      REAL ZZ,UB,LB
      INTEGER COUNT,FLAG,AX,X,Y,NCELLS,NUM,N,DISTR,UBCOUNT,LBCOUNT
      INTEGER*3 TIME
      CHARACTER DISREP*10,ANSB*1
      COUNT=0
      AX=0
      TOTAL=0.0
      CALL STIME
      Z(0,1)=-1.0E30
C
C -----------------------------------------------------------------
C
C       SIMULATE ALL POSSIBLE COMBINATIONS OF THE FUNCTION Z
C
C -----------------------------------------------------------------
C
      DO 10 X=1,NCELLS(1)
         DO 20 Y=1,NCELLS(2)
            AX=AX+1
            CALL FUNCTION (B(X,1),B(Y,2),ZZ)
            CALL SORT (ZZ,Z,COUNT,AX)
            TOTAL=TOTAL+ZZ
20       CONTINUE
10    CONTINUE
C
C -----------------------------------------------------------------
C
C                    END OF THE SIMULATION
C
C -----------------------------------------------------------------
C
      CALL WTIME
      CALL WTIME2 (TIME)
      WRITE (3,*) TIME
      ZMEAN=TOTAL/FLOAT(NCELLS(1)*NCELLS(2))
C
C -----------------------------------------------------------------
C
C           CALCULATE THE JOINT PROBABILITY DISTRIBUTION FUNCTION
C
C  ----------------------------------------------------------------
C
      CALL DISTRIBUTION (Z,COUNT,ZMEAN,ZVARIANCE,NCELLS(1),
```

```
*                       NCELLS(2))
      RETURN
      END
```

```
      SUBROUTINE BOOKBOUND
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE BOOKBOUND                                      *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 4, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE BOOK SIMULATES THE JOINT DISTRIBUTION FUNCTION      *
C *          OF THE TWO RANDOM VARIABLES.                                   *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINES DISTRIBUTION          *
C *                                                FUNCTION                 *
C *                                                SORT                     *
C *                                                                         *
C * USAGE: CALL BOOK                                                        *
C *                                                                         *
C * DISCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *           BOUND(3,2) - REAL - ARRAY BOUND IS A TEMPORARY VARIABLE       *
C *                  THAT COMPUTES THE MINIMUM AND MAXIMUM                  *
C *                  COMBINATIONS OF THE TWO CELLS REPRESENTING THE         *
C *                  TWO RANDOM VARIABLES.                                  *
C *           B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF          *
C *                  GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILITY         *
C *                  PROBABILITY DENSITY FUNCTION.                          *
C *           LB(NCELLS( ),2) - REAL - CONTAINS THE LOWER BOUND             *
C *                  LOCATION FOR CELL NCELLS( ).                           *
C *           NCELLS(NUM) -  INTEGER - THE NUMBER OF DISCRETE               *
C *                  LOCATIONS USED TO APPROXIMATE THE PROBABILITY          *
C *                  DENSITY FUNCTION OF RANDOM VARIABLE NUM.               *
C *           NUM - INTEGER - VARIABLE NUM IS THE NUMBER OF THE             *
C *                  RANDOM VARIABLE.                                       *
C *           UB(NCELLS( ),2) - REAL - CONTAINS THE UPPER BOUND             *
C *                  LOCATION FOR CELL NCELLS( ).                           *
C *     OUTPUT ARGUMENTS:                                                   *
C *           COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z             *
C *                  LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION        *
C *                  FUNCTION.                                              *
C *           LBCOUNT - INTEGER - LBCOUNT IS THE NUMBER OF UNIQUE           *
C *                  LB( , ) LOCATIONS OF THE LOWER BOUND                   *
C *                  DISTRIBUTION FUNCTION.                                 *
C *           UBCOUNT - INTEGER - UBCOUNT IS THE NUMBER OF UNIQUE           *
C *                  UB( , ) LOCATIONS OF THE UPPER BOUND                   *
C *                  DISTRIBUTION FUNCTION.                                 *
C *           TIME - INTEGER*3 - VARIABLE TIME IS THE TOTAL CPU TIME        *
C *                  OF SIMULATION.                                         *
```

```
C *            Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT              *
C *                   PROBABILITY DISTRIBUTION FUNCTION.                 *
C *            Z(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS       *
C *                   THAN OR EQUAL TO Z(COUNT,1).                       *
C *            Z(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF      *
C *                   TIMES Z(COUNT,1) WAS REPEATED DURINC THE           *
C *                   SIMULATION.                                        *
C *            ZLB(COUNT,1) - REAL - THE Z VALUE OF THE LOWER BOUND      *
C *                   OF THE JOINT PROBABILITY DISTRIBUTION FUNCTION.    *
C *            ZLB(COUNT,2) - REAL - THE PROBABILITY OF Z BEINC LESS     *
C *                   THAN OR EQUAL TO ZLB(COUNT,1).                     *
C *            ZLB(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF    *
C *                   TIMES ZLB(COUNT,1) WAS REPEATED DURING THE         *
C *                   SIMULATION.                                        *
C *            ZMEAN - REAL - THE MEAN OF THE JOINT PROBABILTIY          *
C *                   DISTRIBUTION FUNCTION.                             *
C *            ZUB(COUNT,1) - REAL - THE Z VALUE OF THE UPPER BOUND      *
C *                   OF THE JOINT PROBABILITY DISTRIBUTION FUNCTION.    *
C *            ZUB(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS     *
C *                   THAN OR EQUAL TO ZUB(COUNT,1).                     *
C *            ZUB(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF    *
C *                   TIMES ZUB(COUNT,1) WAS REPEATED DURINC THE         *
C *                   SIMULATION.                                        *
C *            ZVARIANCE - REAL - THE VARIANCE OF THE JOINT              *
C *                   PROBABILITY DISTRIBUTION FUNCTION.                 *
C *                                                                      *
C ************************************************************************
C
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      DIMENSION BOUND(3,3)
      REAL B,Z,SX,F,D,WORK,A,VARIANCE,MEAN,ZMEAN,ZVARIANCE,TOTAL
      REAL ZZ,UB,LB,UBTOTAL,LBTOTAL,UBMEAN,LBMEAN,UBVARIANCE
      REAL LBVARIANCE
      INTEGER COUNT,FLAG,AX,X,Y,NCELLS,NUM,N,DISTR,UBCOUNT,LBCOUNT
      INTEGER LBX,UBX
      INTECER*3 TIME
      CHARACTER DISREP*10,ANSB*1
      COUNT=0
      UBCOUNT=0
      LBCOUNT=0
      AX=0
      TOTAL=0.0
      UBTOTAL=0.0
      LBTOTAL=0.0
      WRITE (3,123)
```

```
123     FORMAT (2X,'START SIMULATION')
        CALL STIME
        Z(0,1)=-1.0E30
C
C ---------------------------------------------------------------------
C
C        SIMULATE ALL POSSIBLE COMBINATIONS OF THE FUNCTION Z
C
C ---------------------------------------------------------------------
C
        DO 10 X=1,NCELLS(1)
           WRITE (3,*) X
           DO 20 Y=1,NCELLS(2)
              AX=AX+1
              LBX=LBX+1
              UBX=UBX+1
              CALL FUNCTION (B(X,1),B(Y,2),ZZ)
              CALL SORT (ZZ,Z,COUNT,AX)
              ZMIN=1.0E38
              ZMAX=-1.0E38
              BOUND(1,1)=UB(X,1)
              BOUND(2,1)=LB(X,1)
              BOUND(3,1)=B(X,1)
              BOUND(1,2)=UB(Y,2)
              BOUND(2,2)=LB(Y,2)
              BOUND(3,2)=B(Y,2)
              DO 8000 K=1,3
                 DO 8010 KK=1,3
                    CALL FUNCTION (BOUND(K,1),BOUND(KK,2),ZT)
                    IF (ZT.LT.ZMIN) THEN
                       ZMIN=ZT
                    END IF
                    IF (ZT.GT.ZMAX) THEN
                       ZMAX=ZT
                    END IF
8010             CONTINUE
8000          CONTINUE
              CALL SORT(ZMIN,ZUB,UBCOUNT,UBX)
              CALL SORT(ZMAX,ZLB,LBCOUNT,LBX)
              TOTAL=TOTAL+ZZ
              UBTOTAL=UBTOTAL+ZMIN
              LBTOTAL=LBTOTAL+ZMAX
20         CONTINUE
10      CONTINUE
C
C ---------------------------------------------------------------------
C
C                       END OF THE SIMULATION
```

```
C
C ------------------------------------------------------------------
C
      CALL WTIME
      CALL WTIME2 (TIME)
      WRITE (3,*) TIME
      WRITE (3,124)
124   FORMAT (2X,'END OF SIMULATION')
      ZMEAN=TOTAL/FLOAT(NCELLS(1)*NCELLS(2))
      UBMEAN=UBTOTAL/FLOAT(NCELLS(1)*NCELLS(2))
      LBMEAN=LBTOTAL/FLOAT(NCELLS(1)*NCELLS(2))
C
C ------------------------------------------------------------------
C
C          CALCULATE THE JOINT PROBABILITY DISTRIBUTION FUNCTIONS
C        FOR THE DISCRETE APPROXIMATION AND THE TWO BOUNDING CURVES.
C
C ------------------------------------------------------------------
C
      CALL DISTRIBUTION (Z,COUNT,ZMEAN,ZVARIANCE,NCELLS(1),
     *                   NCELLS(2))
      CALL DISTRIBUTION (ZLB,LBCOUNT,LBMEAN,LBVARIANCE,NCELLS(1),
     *                   NCELLS(2))
      CALL DISTRIBUTION (ZUB,UBCOUNT,UBMEAN,UBVARIANCE,NCELLS(1),
     *                   NCELLS(2))
      RETURN
      END
```

```
      SUBROUTINE CELLR
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE CELLR                                          *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 3, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE CELLR COMPUTES THE CENTER OF GRAVITY LOCATIONS      *
C *          FOR A GIVEN CELL WITH A KNOWN PROBABILITY DISTRIBUTION.        *
C *                                                                         *
C * USAGE: CALL CELLR                                                       *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * DESCRIPTION OF PRAMETERS:                                               *
C *     INPUT ARGUMENTS:                                                    *
C *             A(N) - REAL - WHERE N=NCELLS*2, ODD VALUES OF N             *
C *                    REPRESENT THE CENTER OF GRAVITY LOCATIONS, AND       *
C *                    THE EVEN VALUES OF N INDICATE THE CELL               *
C *                    BOUNDARIES.                                          *
C *             ANSB - CHARACTER*1 - IF ANSB IS EQUAL TO 'Y' THEN THE       *
C *                    PROGRAM COMPUTES THE UPPER AND LOWER BOUNDS OF       *
C *                    THE JOINT DISTRIBUTION FUNCTION AS WELL AS THE       *
C *                    DISCRETE APPROXIMATION FUNCTION.  IF ANSB IS         *
C *                    EQUAL TO 'N' THEN THE PROGRAM COMPUTES ONLY THE      *
C *                    DISCRETE APPROXIMATION OF THE JOINT                  *
C *                    DISTRIBUTION FUNCTION.                               *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                    LOCATIONS THAT APPROXIMATE THE PROBABILITY           *
C *                    DENSITY FUNCTION OF RANDOM VARIABLE NUM.             *
C *             NUM - INTEGER - VARIABLE NUM IS THE NUMBER OF THE           *
C *                    RANDOM VARIABLE.                                     *
C *                                                                         *
C *     OUTPUT ARGUMENTS:                                                   *
C *             B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF        *
C *                    GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILITY       *
C *                    DENSITY FUNCTION.                                    *
C *             LB(NCELLS( ),2) - REAL - CONTAINS THE LOWER BOUND           *
C *                    LOCATION FOR CELL NCELLS( ).                         *
C *             UB(NCELLS( ),2) - REAL - CONTAINS THE UPPER BOUND           *
C *                    LOCATION FOR CELL NCELLS( ).                         *
C *     OUTPUT ARGUMENTS:                                                   *
C *             COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z           *
C *                    LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION      *
C *                    FUNCTION.                                            *
C *                                                                         *
```

```
C ***********************************************************************
C
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      INTEGER BX,NCELLS,NUM,COUNT,DISTR,UBCOUNT,LBCOUNT
      REAL A,B,Z,VARIANCE,MEAN,ZMEAN,ZVARIANCE,UB,LB
      CHARACTER DISREP*10,ANSB*1
      WRITE (3,2)
2     FORMAT (2X,'ENTERED CELLR')
      BX=0
      DO 80 K=1,NCELLS(NUM)*2,2
         BX=BX+1
         B(BX,NUM)=A(K)
80    CONTINUE
C
C -------------------FOR BOUNDING-------------------------------
C
      IF (ANSB.EQ.'Y') THEN
         BX=1
         DO 10 I=0,NCELLS(NUM)*2,2
            LB(BX,NUM)=A(I)
            UB(BX,NUM)=A(I+2)
            BX=BX+1
10       CONTINUE
      END IF
C
C ------------------END FOR BOUNDING---------------------------
C
      RETURN
      END
```

```
      SUBROUTINE DIST
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE DIST                                           *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 3, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE DIST INITIALIZES THE TYPE OF PROBABILITY            *
C *          DISTRIBUTION FOR EACH RANDOM VARIABLE.                         *
C *                                                                         *
C * USAGE: CALL DIST                                                        *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINES CELLR                 *
C *                                                   NORMAL                *
C *                                                   NORMINV               *
C *                                                   TRIANGEL              *
C *                                                   UNIFORM               *
C *                                                   SPECIAL               *
C *                                                                         *
C * DESCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             DISTR(NUM) - INTEGER - THE NUMBER OF THE DISTRIBUTION       *
C *                    FOR RANDOM VARIABLE NUM (1=NORMAL, 2=UNIFORM,        *
C *                     3=TRIANGULAR, 4=SPECIAL).                           *
C *             NUM - INTEGER - THE NUMBER OF THE RANDOM VARIABLE.          *
C *                                                                         *
C *     OUTPUT ARGUMENTS:                                                   *
C *             A(N) - REAL - WHERE N=NCELLS*2, ODD VALUES OF N             *
C *                    REPRESENT THE CENTER OF GRAVITY LOCATIONS AND        *
C *                    EVEN VALUES OF N INDICATE THE CELL BOUNDARIES.       *
C *             DISREP - CHARACTER*10 - DISREP CONTAINS THE TYPE OF         *
C *                    PROBABILITY DISTRIBUTION FOR VARIABLE NUM            *
C *                    (NORMAL, UNIFORM, TRIANGULAR, SPECIAL).              *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                    LOCATIONS THAT APPROXIMATE THE PROBABILITY           *
C *                    DENSITY FUNCTION OF RANDOM VARIABLE NUM.             *
C *                                                                         *
C ***************************************************************************
C
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      INTEGER DISTR,NUM,NCELLS,COUNT,LBCOUNT,UBCOUNT
      REAL VARIANCE,MEAN,A,B,Z,ZMEAN,ZVARIANCE,UB,LB
      CHARACTER DISREP*10,ANSB*1
```

```
5       WRITE (3,*)
        WRITE (3,10)
10      FORMAT (2X,'DISTRIBUTIONS')
        WRITE (3,20)
20      FORMAT (2X,'1)  NORMAL')
        WRITE (3,30)
30      FORMAT (2X,'2)  UNIFORM')
        WRITE (3,40)
40      FORMAT (2X,'3)  TRIANGULAR')
        WRITE (3,50)
50      FORMAT (2X,'4)  SPECIAL')
        WRITE (3,80) NUM
80      FORMAT (2X,'ENTER NUMBER OF DISTRIBUTION FOR VARIABLE',I3)
        READ (0,*) DISTR(NUM)
        IF (DISTR(NUM).GT.4) GOTO 5
        IF (DISTR(NUM).EQ.1) THEN
           DISREP(NUM)='NORMAL'
           CALL NORMAL
        ELSE
           IF (DISTR(NUM).EQ.2) THEN
              DISREP(NUM)='UNIFORM'
              CALL UNIFORM
           ELSE
              IF (DISTR(NUM).EQ.3) THEN
                 DISREP(NUM)='TRIANGULAR'
                 CALL TRIANGLE
              ELSE
                 IF (DISTR(NUM).EQ.4) THEN
                    DISREP(NUM)='SPECIAL'
                    CALL SPECIAL
                 ELSE
                    GOTO 5
                 END IF
              END IF
           END IF
        END IF
        RETURN
        END
```

```
      SUBROUTINE DISTRIBUTION (Z,COUNT,MEAN,VARIANCE,NC1,NC2)
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE DISTRIBUTION                                   *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 23, 1987                                                    *
C *                                                                         *
C * PURPOSE: SUBROUTINE DISTRIBUTION COMPUTES THE JOINT DISTRIBUTION        *
C *          FUNCTION OF RANDOM VARIABLE Z.                                 *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * USAGE: CALL DISTRIBUTION(Z,COUNT,MEAN,VARIANCE,NC1,NC2)                 *
C *                                                                         *
C * DESCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *           COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z             *
C *                   LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION       *
C *                   FUNCTION.                                             *
C *           MEAN - REAL - THE MEAN OF THE JOINT PROBABILITY               *
C *                   DISTRIBUTION FUNCTION.                                *
C *           NC1 - INTEGER - THE NUMBER OF DISCRETE LOCATIONS THAT         *
C *                   APPROXIMATE THE PROBABILTIY DENSITY FUNCTION          *
C *                   OF RANDOM VARIABLE 1.                                 *
C *           NC2 - INTEGER - THE NUMBER OF DISCRETE LOCATIONS THAT         *
C *                   APPROXIMATE THE PROBABILTIY DENSITY FUNCTION          *
C *                   OF RANDOM VARIABLE 2.                                 *
C *           Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT                  *
C *                   PROBABILITY DISTRIBUTION FUNCTION.                    *
C *           Z(COUNT,2) - REAL - THE NUMBER OF TIMES THE VALUE OF          *
C *                   TIMES Z(COUNT,1) WAS REPEATED DURING THE              *
C *                   SIMULATION.                                           *
C *                                                                         *
C *     OUTPUT ARGUMENTS:                                                   *
C *           VARIANCE - REAL - THE VARIANCE OF THE JOINT PROBABILITY       *
C *                   DISTRIBUTION FUNCTION.                                *
C *           Z(COUNT,3) - REAL - THE PROBABILITY OF Z BEING LESS           *
C *                   THAN OR EQUAL TO Z(COUNT,1).                          *
C *                   TIMES Z(COUNT,1) WAS REPEATED DURING THE              *
C *                   SIMULATION.                                           *
C *                                                                         *
C ***************************************************************************
C
      DIMENSION Z(0:40000,3)
      REAL Z,ZT,MEAN,VARIANCE
      INTEGER COUNT,NC1,NC2
```

```
      ZT=0.0
      VARIANCE=0.0
      DO 80 I=1,COUNT
         Z(I,3)=Z(I,2)
         ZT=ZT+Z(I,3)
         Z(I,2)=ZT/FLOAT(NC1*NC2)
         VARIANCE=(((Z(I,1)-MEAN)**2.0)*(Z(I,3)/FLOAT(NC1*NC2)))+
     *           VARIANCE
80    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FUNCTION (X1,X2,ANSZ)
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE FUNCTION                                       *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 23, 1987                                                    *
C *                                                                         *
C * PURPOSE: SUBROUTINE FUNCTION COMPUTES THE VALUE OF ANSZ GIVEN           *
C *          THE VARIABLES X1 AND X2.                                       *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * USAGE: CALL FUNCTION (X1,X2,ANSZ)                                       *
C *                                                                         *
C * DISCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             X1 - REAL - THE VALUE OF THE FIRST VARIABLE.                *
C *             X2 - REAL - THE VALUE OF THE SECOND VARIABLE.               *
C *     OUTPUT ARGUMENTS:                                                   *
C *             ANSZ - REAL - THE VALUE OF THE FUNCTION GIVEN THE           *
C *                    VALUES OF THE TWO RANDOM VARIABLES X1 AND X2.        *
C *                                                                         *
C ***************************************************************************
C
      REAL X1,X2,ANSZ
      PI=3.141592654
C
C ---------------------------------------------------------------------
C
C                  ENTER FUNCTION TO BE SIMULATED
C                       X1 = RANDOM VARIABLE 1
C                       X2 = RANDOM VARIABLE 2
C                       ANSZ = JOINT RANDOM VARIABLE
C
C ---------------------------------------------------------------------
C
C     ANSZ=(X1/2.0)+X2
C     ANSZ=SQRT(X1**2+X2**2)
C     ANSZ=X1*X2*12.0
C     ANSZ=2.5-(X1*X2)
C     ANSZ=(X1/2.0+X2/2.0)
C     ANSZ=(X1*X2**2.0)/2.0
C     ANSZ=(X1*(X2**3.0))/6.0
C     ANSZ=X1*(X2**2.0/8.0)
C     ANSZ=X1*(1/X2)
      ANSZ=X1-((4*50000)/(3.141592654*X2**2))
```

```
C       ANSZ=X1/(1+X2)
C       ANSZ=LOG10((1+X1)/X2)
C       ANSZ=X1-((4.0*50000)/(PI*X2))
C       ANSZ=SQRT(X1**2.+X2**2.)
C
C -------------------------------------------------------------------
C
        RETURN
        END
```

```
      SUBROUTINE NORMAL
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE NORMAL                                         *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 3, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE NORMAL FINDS THE NCELL LOCATION ON THE X AXIS       *
C *          GIVEN A PROBABILITY (AREA) FOR A NORMAL RANDOM VARIABLE.       *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINES CELLR                 *
C *                                                    NORMINV              *
C *                                                                         *
C * USAGE: CALL NORMAL                                                      *
C *                                                                         *
C * DISCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             MEAN(NUM) - REAL - THE MEAN VALUE FOR RANDOM VARIABLE       *
C *                 NUM.                                                    *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                 LOCATIONS THAT APPROXIMATE THE PROBABILITY              *
C *                 DENSITY FUNCTION OF RANDOM VARIABLE NUM.                *
C *             NUM - INTEGER - THE NUMBER OF THE RANDOM VARIABLE.          *
C *             VARIANCE - REAL - THE VARIANCE VALUE FOR RANDOM             *
C *                 VARIABLE NUM.                                           *
C *                                                                         *
C *     OUTPUT ARGUMENTS:                                                   *
C *             A(N) - REAL - WHERE N=NCELLS*2, ODD VALUES OF N             *
C *                 REPRESENT THE CENTER OF GRAVITY LOCATIONS AND           *
C *                 THE EVEN VALUES OF N INDICATE THE CELL                  *
C *                 BOUNDARIES.                                             *
C *             B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF        *
C *                 GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILITY          *
C *                 DENSITY FUNCTION.                                       *
C *                                                                         *
C ***************************************************************************
C
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      REAL AREA,A,X,PROB,KAREA,MEAN,VARIANCE,B,Z
      REAL ZMEAN,ZVARIANCE,UB,LB
      INTEGER NCELLS,K,NUM,COUNT,DISTR,UBCOUNT,LBCOUNT
      CHARACTER DISREP*10,ANSB*1
      WRITE (3,*)
```

```
      WRITE (3,*)
      WRITE (3,20) NUM
20    FORMAT (2X,'NORMAL DISTRIBUTION FOR VARIABLE',I3)
      WRITE (3,30)
30    FORMAT (2X,'ENTER MEAN')
      READ (0,*) MEAN(NUM)
      WRITE (3,40)
40    FORMAT (2X,'ENTER STANDARD DEVIATION')
      READ (0,*) VARIANCE(NUM)
      VARIANCE(NUM)=VARIANCE(NUM)**2.0
      WRITE (3,3) NUM
3     FORMAT (2X,'ENTER NUMBER OF CELLS FOR VARIABLE',I3)
      READ (0,*) NCELLS(NUM)
      AREA=1/(FLOAT(NCELLS(NUM))*2.0)
      DO 10 K=1,NCELLS(NUM)*2.0
         KAREA=AREA*(FLOAT(K))
         P=KAREA
         CALL NORMINV(P,X,D,IE)
         A(K)=X*SQRT(VARIANCE(NUM))+MEAN(NUM)
10    CONTINUE
      A(0)=-A(NCELLS(NUM)*2)
      CALL CELLR
      RETURN
      END
```

```
      SUBROUTINE NORMINV (P,X,D,IE)
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: NORMINV                                                   *
C *                                                                         *
C * PURPOSE: SUBROUTINE NORMINV WAS SUPPLIED FROM THE SCIENTIFIC            *
C *          SUBROUTINE PACKAGE (SSP).  THE SSP CONTAINS A COLLECTION       *
C *          OF MATHEMATICAL AND STATISTICAL ROUTINES FOR THE               *
C *          HARRIS COMPUTER SYSTEM.                                        *
C *                                                                         *
C ***************************************************************************
C
C.....................................................................
C
C       SUBROUTINE NORMINV
C
C       PURPOSE
C         COMPUTES X = P**(-1)(Y), THE ARGUMENT X SUCH THAT Y= P(X) =
C         THE PROBABILITY THAT THE RANDOM VARIABLE U, DISTRIBUTED
C         NORMALLY(0,1), IS LESS THAN OR EQUAL TO X.  F(X), THE
C         ORDINATE OF THE NORMAL DENSITY, AT X, IS ALSO COMPUTED.
C
C       USAGE
C         CALL NORMINV (P,X,D,IER)
C
C       DESCRIPTION OF PARAMETERS
C         P   - INPUT PROBABILITY.
C         X   - OUTPUT ARGUMENT SUCH THAT P = Y = THE PROBABILITY THAT
C                 U, THE RANDOM VARIABLE, IS LESS THAN OR EQUAL TO X.
C         D   - OUTPUT DENSITY, F(X).
C         IER - OUTPUT ERROR CODE
C               = -1 IF P IS NOT IN THE INTERVAL (0,1), INCLUSIVE.
C                 X=D=.99999E+38 IN THIS CASE
C               = 0 IF THERE IS NO ERROR.  SEE REMARKS, BELOW.
C
C       REMARKS
C         MAXIMUM ERROR IS 0.00045.
C         IF P = 0, X IS SET TO -(10)**38.  D IS SET TO 0.
C         IF P = 1, X IS SET TO  (10)**38.  D IS SET TO 0.
C
C       SUBROUTINES AND SUBPROGRAMS REQUIRED
C         NONE
C
C       METHOD
C         BASED ON APPROXIMATIONS IN C. HASTINGS, APPROXIMATIONS FOR
C         DIGITAL COMPUTERS, PRINCETON UNIV. PRESS, PRINCETON, N.J.,
C         1955.  SEE EQUATION 26.2.23, HANDBOOK OF MATHEMATICAL
C         FUNCTIONS, ABRAMOWITZ AND STEGUN, DOVER PUBLICATIONS, INC.,
```

```
C          NEW YORK.
C
C.....................................................................
C
C
      REAL X,D,P,T2,T
      INTEGER IE
      IE=0
      X=1.7E+38
      D=X
      IF(P)1,4,2
    1 IE=-1
      GO TO 12
    2 IF (P-1.0)7,5,1
    4 X=-1.7E+38
    5 D=0.0
      GO TO 12
C
C
    7 D=P
      IF(D-0.5)9,9,8
    8 D=1.0-D
    9 T2=ALOG(1.0/(D*D))
      T=SQRT(T2)

      X=T-(2.515517+0.802853*T+0.010328*T2)/(1.0+1.432788*T+0.189269*T2
     1  +0.001308*T*T2)
      IF(P-0.5)10,10,11
   10 X=-X
   11 D=0.3989423*EXP(-X*X/2.0)
   12 RETURN
      END
```

```
      SUBROUTINE OUTPUT
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE OUTPUT                                         *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 3, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE OUTPUT PRINTS THE CENTER OF GRAVITY LOCATIONS       *
C *          FOR RANDOM VARIABLES A AND B, AND THE JOINT PROBABILITY        *
C *          DISTRIBUTION OF VARIABLE Z TO FILE W2.  OUTPUT ALSO PRINTS     *
C *          THE JOINT DISTRIBUTION FUNCTION TO A GIVEN FILE SO THE         *
C *          JOINT DISTRIBUTION CAN BE INPUT ACAIN AS A 'SPECIAL'           *
C *          DISTRIBUTION FUNCTION.                                         *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * USAGE: CALL OUTPUT                                                      *
C *                                                                         *
C * DESCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF        *
C *                     GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILTIY      *
C *                     DENSITY FUNCTION.                                   *
C *             COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z           *
C *                     LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION     *
C *                     FUNCTION.                                           *
C *             DISREP(NUM) - CHARACTER*10 - DISREP CONTAINS THE            *
C *                     TYPE OF PROBABILITY DISTRIBUTION FOR VARIABLE       *
C *                     NUM (NORMAL, UNIFORM, TRIANGULAR, SPECIAL).         *
C *             MEAN(NUM) - REAL - THE MEAN VALUE FOR RANDOM VARIABLE       *
C *                     NUM.                                                *
C *             NAME2 - CHARACTER*30 - NAME2 IS THE OUTPUT DATA FILE        *
C *                     NAME OF THE JOINT DISTRIBUTION FUNCTION.            *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                     LOCATIONS THAT APPROXIMATE THE PROBABILITY          *
C *                     DENSITY FUNCTION OF RANDOM VARIABLE NUM.            *
C *             NUM - INTEGER - RANGE OF 1 TO 2, VARIABLE 'NUM' IS THE      *
C *                     NUMBER OF THE RANDOM VARIABLE.                      *
C *             VARIANCE(NUM) - REAL - THE VARIANCE VALUE FOR RANDOM        *
C *                     VARIABLE NUM.                                       *
C *             Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT                *
C *                     PROBABILITY DISTRIBUTION FUNCTION.                  *
C *             Z(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS         *
C *                     THAN OR EQUAL TO Z(COUNT,1).                        *
C *             Z(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF        *
C *                     TIMES Z(COUNT,1) WAS REPEATED DURING THE            *
```

```
C *                    SIMULATION.                                        *
C *              ZMEAN - REAL - THE MEAN OF THE JOINT PROBABILITY         *
C *                    DISTRIBUTION FUNCTION.                             *
C *              ZVARIANCE - REAL - THE VARIANCE OF THE JOINT             *
C *                    PROBABILITY DISTRIBUTION FUNCTION.                 *
C *                                                                       *
C *     OUTPUT ARGUMENTS:                                                 *
C *              ZSTANDARD_DEV - REAL - THE STANDARD DEVIATION OF THE     *
C *                    JOINT PROBABILITY DISTRIBUTION Z.                  *
C *                                                                       *
C ***************************************************************************
C
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      REAL Z,MAXDEV,B,MEAN,VARIANCE,A,ZMEAN,ZVARIANCE,UB,LB
      INTEGER NCELLS,COUNT,DISTR,NUM,UBCOUNT,LBCOUNT
      CHARACTER DISREP*10,NAME*20,NAME2*20,ANSB*1
C
C ---------------------------------------------------------------------
C
C                     PRINT DATA TO FILE "W2"
C
C ---------------------------------------------------------------------
C
      WRITE (23,*)
      WRITE (23,*)
      WRITE (23,70)
70    FORMAT (2X,'RANDOM VARIABLE: 1',18X,'RANDOM VARIABLE: 2')
      WRITE (23,80) DISREP(1),DISREP(2)
80    FORMAT (2X,'DISTRIBUTION: ',A11,11X,'DISTRIBUTION:  ',A)
      WRITE (23,90) MEAN(1),MEAN(2)
90    FORMAT (2X,'MEAN: ',F12.6,18X,'MEAN: ',F12.6)
      WRITE (23,100) VARIANCE(1),VARIANCE(2)
100   FORMAT (2X,'VARIANCE: ',F12.6,14X,'VARIANCE: ',F12.6)
      STANDARD_DEV1=SQRT(VARIANCE(1))
      STANDARD_DEV2=SQRT(VARIANCE(2))
      WRITE (23,101) STANDARD_DEV1,STANDARD_DEV2
101   FORMAT (2X,'STANDARD DEVIATION: ',F12.6,4X,
     *'STANDARD DEVIATION: ',F12.6)
      WRITE (23,170) NCELLS(1),NCELLS(2)
170   FORMAT (2X,'NUMBER OF CELLS :',I4,15X,'NUMBER OF CELLS: ',I4)
      WRITE (23,*)
      WRITE (23,110)
110   FORMAT (6X,'X LOCATION',26X,'X LOCATION')
      WRITE (23,120)
120   FORMAT (4X,'FOR VARIABLE 1',22X,'FOR VARIABLE 2')
```

```
      WRITE (23,113)
113   FORMAT (4X,'--------------',22X,'--------------')
      IF (NCELLS(1).GT.NCELLS(2)) THEN
         INCELLS=NCELLS(1)
      ELSE
         INCELLS=NCELLS(2)
      END IF
      DO 160 I=1,INCELLS
         IF (I.LE.NCELLS(1).AND.I.LE.NCELLS(2)) THEN
            WRITE (23,130) B(I,1),B(I,2)
130         FORMAT (4X,F12.6,23X,F12.6)
         ELSE
            IF (I.LE.NCELLS(1)) THEN
               WRITE (23,140) B(I,1)
140            FORMAT (4X,F12.6)
            ELSE
               WRITE (23,150) B(I,2)
150            FORMAT (37X,F12.6)
            END IF
         END IF
160   CONTINUE
      WRITE (23,*)
      WRITE (23,161)
161   FORMAT (19X,'****** Z FUNCTION ******')
      WRITE (23,11) COUNT
11    FORMAT (15X,'NUMBER OF UNIQUE Z LOCATIONS: ',I5)
      WRITE (23,34) ZMEAN
34    FORMAT(15X,'MEAN: ',F12.6)
      WRITE (23,162) ZVARIANCE
162   FORMAT (15X,'VARIANCE: ',F12.6)
      ZSTANDARD_DEV=SQRT(ZVARIANCE)
      WRITE (23,163) ZSTANDARD_DEV
163   FORMAT (15X,'STANDARD DEVIATION: ',F12.6)
      WRITE (23,*)
      WRITE (23,13)
13    FORMAT (15X,'CELL')
      WRITE (23,12)
12    FORMAT (16X,'NO.',3X,'Z LOCATION',7X,'Z DISTRIBUTION')
      WRITE (23,132)
132   FORMAT (15X,'----',3X,'----------',7X,'--------------')
      DO 10 I=1,COUNT
         WRITE (23,20) I,Z(I,1),Z(I,2)
20       FORMAT (14X,I4,2X,F12.6,6X,F12.6)
10    CONTINUE
C
C -----------------------------------------------------------------------
C
C                  PRINT DISTRIBUTION DATA TO NAME2 FILE
```

```
C
C ---------------------------------------------------------------------
C
      WRITE (3,116)
116   FORMAT (2X,'ENTER DISTRIBUTION DATA FILE NAME')
      READ (0,117) NAME2
117   FORMAT (A20)
      OPEN (UNIT=25,FILE=NAME2)
      WRITE (25,538) NCELLS(1),NCELLS(2)
538   FORMAT (I5,I5)
      WRITE (25,189) COUNT
189    FORMAT (I5)
      DO 89 I=1,COUNT
         WRITE (25,112) Z(I,1),Z(I,2),Z(I,3)
112      FORMAT (F20.7,F12.7,F12.7)
89    CONTINUE
      CLOSE 25
30    RETURN
      END
```

```
      SUBROUTINE OUTPUTBOUND
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE OUTPUTBOUND                                    *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 3, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE OUTPUT PRINTS THE CENTER OF GRAVITY LOCATIONS       *
C *          FOR RANDOM VARIABLES A AND B, AND THE JOINT PROBABILITY        *
C *          DISTRIBUTION OF VARIABLE Z TO FILE W2.  OUTPUT ALSO PRINTS     *
C *          THE JOINT DISTRIBUTION FUNCTION TO A GIVEN FILE SO THE         *
C *          JOINT DISTRIBUTION CAN BE INPUT AGAIN AS A 'SPECIAL'           *
C *          DISTRIBUTION FUNCTION.                                         *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * USAGE: CALL OUTPUT                                                      *
C *                                                                         *
C * DESCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF        *
C *                     GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILTIY      *
C *                     DENSITY FUNCTION.                                   *
C *             COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z           *
C *                     LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION     *
C *                     FUNCTION.                                           *
C *             DISREP(NUM) - CHARACTER*10 - DISREP CONTAINS THE            *
C *                     TYPE OF PROBABILITY DISTRIBUTION FOR VARIABLE       *
C *                     NUM (NORMAL, UNIFORM, TRIANGULAR, SPECIAL).         *
C *             MEAN(NUM) - REAL - THE MEAN VALUE FOR RANDOM VARIABLE       *
C *                     NUM.                                                *
C *             NAME2 - CHARACTER*30 - NAME2 IS THE OUTPUT DATA FILE        *
C *                     NAME OF THE JOINT DISTRIBUTION FUNCTION.            *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                     LOCATIONS THAT APPROXIMATE THE PROBABILITY          *
C *                     DENSITY FUNCTION OF RANDOM VARIABLE NUM.            *
C *             NUM - INTEGER - RANGE OF 1 TO 2, VARIABLE 'NUM' IS THE      *
C *                     NUMBER OF THE RANDOM VARIABLE.                      *
C *             VARIANCE(NUM) - REAL - THE VARIANCE VALUE FOR RANDOM        *
C *                     VARIABLE NUM.                                       *
C *             Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT                *
C *                     PROBABILITY DISTRIBUTION FUNCTION.                  *
C *             Z(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS         *
C *                     THAN OR EQUAL TO Z(COUNT,1).                        *
C *             Z(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF        *
C *                     TIMES Z(COUNT,1) WAS REPEATED DURING THE            *
```

```
C *                   SIMULATION.                                          *
C *             ZLB(COUNT,1) - REAL - THE Z VALUE OF THE LOWER BOUND       *
C *                   OF THE JOINT PROBABILITY DISTRIBUTION FUNCTION.      *
C *             ZLB(COUNT,2) - REAL - THE PROBABILITY OF Z BEINC LESS      *
C *                   THAN OR EQUAL TO ZLB(COUNT,1).                       *
C *             ZLB(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF     *
C *                   TIMES ZLB(COUNT,1) WAS REPEATED DURING THE           *
C *                   SIMULATION.                                          *
C *             ZMEAN - REAL - THE MEAN OF THE JOINT PROBABILITY           *
C *                   DISTRIBUTION FUNCTION.                               *
C *             ZUB(COUNT,1) - REAL - THE Z VALUE OF THE UPPER BOUND       *
C *                   OF THE JOINT PROBABILITY DISTRIBUTION FUNCTION.      *
C *             ZUB(COUNT,2) - REAL - THE PROBABILITY OF Z BEINC LESS      *
C *                   THAN OR EQUAL TO ZUB(COUNT,1).                       *
C *             ZUB(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF     *
C *                   TIMES ZUB(COUNT,1) WAS REPEATED DURINC THE           *
C *                   SIMULATION.                                          *
C *             ZVARIANCE - REAL - THE VARIANCE OF THE JOINT               *
C *                   PROBABILITY DISTRIBUTION FUNCTION.                   *
C *                                                                        *
C *     OUTPUT ARCUMENTS:                                                  *
C *             ZSTANDARD_DEV - REAL - THE STANDARD DEVIATION OF THE       *
C *                   JOINT PROBABILITY DISTRIBUTION Z.                    *
C *                                                                        *
C ************************************************************************
C
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      REAL Z,MAXDEV,B,MEAN,VARIANCE,A,ZMEAN,ZVARIANCE,UB,LB
      INTECER NCELLS,COUNT,DISTR,NUM,UBCOUNT,LBCOUNT
      CHARACTER DISREP*10,NAME*20,NAME2*20,ANSB*1
C
C ---------------------------------------------------------------------
C
C                   PRINT DATA TO FILE "W2"
C
C ---------------------------------------------------------------------
C
      WRITE (23,70)
70    FORMAT (14X,'RANDOM VARIABLE: 1',34X,'RANDOM VARIABLE: 2')
      WRITE (23,80) DISREP(1),DISREP(2)
80    FORMAT (14X,'DISTRIBUTION: ',A11,27X,'DISTRIBUTION:  ',A)
      WRITE (23,90) MEAN(1),MEAN(2)
90    FORMAT (14X,'MEAN: ',F12.6,34X,'MEAN: ',F12.6)
      WRITE (23,100) VARIANCE(1),VARIANCE(2)
100   FORMAT (14X,'VARIANCE: ',F12.6,30X,'VARIANCE: ',F12.6)
```

```
      STANDARD_DEV1=SQRT(VARIANCE(1))
      STANDARD_DEV2=SQRT(VARIANCE(2))
      WRITE (23,101) STANDARD_DEV1,STANDARD_DEV2
101   FORMAT (14X,'STANDARD DEVIATION: ',F12.6,20X,
     *'STANDARD DEVIATION: ',F12.6)
      WRITE (23,170) NCELLS(1),NCELLS(2)
170   FORMAT (14X,'NUMBER OF CELLS :',I4,31X,'NUMBER OF CELLS: ',I4)
      WRITE (23,*)
      WRITE (23,110)
110   FORMAT (8X,'X1 MINIMUM',7X,'CENTER OF',6X,'X1 MAXIMUM',
     * 10X,'X2 MINIMUM',7X,'CENTER OF',6X,'X2 MAXIMUM')
      WRITE (23,120)
120   FORMAT (11X,'VALUE',10X,'GRAVITY',10X,'VALUE',15X,'VALUE',
     * 10X,'GRAVITY',10X,'VALUE')
      WRITE (23,123)
123   FORMAT (6X,'------------',4X,'------------',4X,'------------'
     *,8X,'------------',4X,'------------',4X,'------------')
      IF (NCELLS(1).GT.NCELLS(2)) THEN
         INCELLS=NCELLS(1)
      ELSE
         INCELLS=NCELLS(2)
      END IF
      DO 160 I=1,INCELLS
         IF (I.LE.NCELLS(1).AND.I.LE.NCELLS(2)) THEN
            WRITE (23,130) LB(I,1),B(I,1),UB(I,1),LB(I,2),B(I,2),
     *                     UB(I,2)
130         FORMAT (6X,F12.7,4X,F12.7,4X,F12.7,8X,F12.7,4X,F12.7,
     *                     4X,F12.7)
         ELSE
            IF (I.LE.NCELLS(1)) THEN
               WRITE (23,140) LB(I,1),B(I,1),UB(I,1)
140            FORMAT (6X,F12.7,4X,F12.7,4X,F12.7)
            ELSE
               WRITE (23,150) LB(I,2),B(I,2),UB(I,2)
150            FORMAT (39X,F12.7,4X,F12.7,4X,F12.7)
            END IF
         END IF
160   CONTINUE
      WRITE (23,*)
      WRITE (23,*)
      WRITE (23,161)
161   FORMAT (38X,'********** Z FUNCTION **********')
      WRITE (23,11) COUNT
11    FORMAT (38X,'NUMBER OF UNIQUE Z LOCATIONS: ',I5)
      WRITE (23,34) ZMEAN
      WRITE (23,162) ZVARIANCE
162   FORMAT (38X,'VARIANCE: ',F12.5)
      ZSTANDARD_DEV=SQRT(ZVARIANCE)
```

```
      WRITE (23,163) ZSTANDARD_DEV
163   FORMAT (38X,'STANDARD DEVIATION: ',F12.5)
34    FORMAT(38X,'MEAN: ',F12.5)
      WRITE (23,*)
      WRITE (23,13)
13    FORMAT (2X,'CELL',6X,'DISCRETE APPROXIMATION',15X,
     * 'LOWER BOUNDS',20X,'UPPER BOUNDS')
      WRITE (23,12)
12    FORMAT (3X,'NO.',9X,'Z',9X'DISTRIBUTION',10X,'Z',9X,
     * 'DISTRIBUTION',10X,'Z',9X,'DISTRIBUTION')
      WRITE (23,2000)
2000  FORMAT (2X,'----',3X,'------------',4X,'------------',4X,
     * '------------',4X,'------------',4X,'------------',4X,
     * '------------')
      ICOUNT=-9999
      IF (UBCOUNT.GT.ICOUNT) THEN
         ICOUNT=UBCOUNT
      ELSE
         IF (LBCOUNT.GT.ICOUNT) THEN
            ICOUNT=LBCOUNT
         ELSE
            ICOUNT=COUNT
         END IF
      END IF
      DO 10 I=1,ICOUNT
         IF (I.LE.UBCOUNT.AND.I.LE.LBCOUNT.AND.I.LE.COUNT) THEN
            WRITE (23,20) I,Z(I,1),Z(I,2),ZLB(I,1),ZLB(I,2),
     *                    ZUB(I,1),ZUB(I,2)
20          FORMAT (1X,I4,3X,F12.7,4X,F12.7,4X,F12.7,4X,F12.7,
     *                    4X,F12.7,4X,F12.7)
         ELSE
            IF (I.LE.LBCOUNT.AND.I.LE.COUNT.AND.I.GT.UBCOUNT) THEN
               WRITE (23,720) I,Z(I,1),Z(I,2),ZLB(I,1),ZLB(I,2)
720            FORMAT (1X,I4,3X,F12.7,4X,F12.7,4X,F12.7,4X,F12.7)
            ELSE
               IF (I.LE.LBCOUNT.AND.I.GT.COUNT.AND.I.LE.UBCOUNT)
     *            THEN
                  WRITE (23,730) I,ZLB(I,1),ZLB(I,2),ZUB(I,1),
     *                 ZUB(I,2)
730               FORMAT (1X,I4,35X,F12.7,4X,F12.7,4X,F12.7
     *                 ,4X,F12.7)
               ELSE
                  IF (I.GT.LBCOUNT.AND.I.LE.COUNT.AND.I.LE.UBCOUNT)
     *                THEN
                     WRITE (23,740) I,Z(I,1),Z(I,2),
     *                      ZUB(I,1),ZUB(I,2)
740                  FORMAT (1X,I4,3X,F12.7,4X,F12.7,36X,F12.7,4X,
     *                      F12.7)
```

```
                  ELSE
                     IF (I.LE.LBCOUNT.AND.I.GT.COUNT.
     *                   AND.I.GT.UBCOUNT) THEN
                         WRITE (23,750) I,ZLB(I,1),ZLB(I,2)
750                      FORMAT (1X,I4,35X,F12.7,4X,F12.7)
                     ELSE
                        IF (I.LE.COUNT.AND.I.GT.LBCOUNT.
     *                      AND.I.GT.UBCOUNT) THEN
                            WRITE (23,760) I,Z(I,1),Z(I,2)
760                         FORMAT (1X,I4,3X,F12.7,4X,F12.7)
                        ELSE
                            WRITE (23,770) I,ZUB(I,1),ZUB(I,2)
770                         FORMAT (1X,I4,67X,F12.7,4X,F12.7)
                        END IF
                     END IF
                  END IF
               END IF
            END IF
10       CONTINUE
C
C ---------------------------------------------------------------------
C
C                  PRINT DISTRIBUTION DATA TO NAME2 FILE
C
C ---------------------------------------------------------------------
C
         WRITE (3,116)
116      FORMAT (2X,'ENTER DISTRIBUTION DATA FILE NAME')
         READ (0,117) NAME2
117      FORMAT (A20)
         OPEN (UNIT=25,FILE=NAME2)
         WRITE (25,538) NCELLS(1),NCELLS(2)
538      FORMAT (I5,I5)
         WRITE (25,189) UBCOUNT,LBCOUNT
189      FORMAT (I5,I5)
         IF (UBCOUNT.GT.LBCOUNT) THEN
            ICOUNT=UBCOUNT
         ELSE
            ICOUNT=LBCOUNT
         END IF
         DO 89 I=1,ICOUNT
            IF (I.LE.LBCOUNT.AND.I.LE.UBCOUNT) THEN
               WRITE (25,112) ZUB(I,1),ZUB(I,2),ZLB(I,1),ZLB(I,2)
112            FORMAT (F12.6,1X,F12.6,1X,F12.6,1X,F12.6)
            ELSE
               IF (I.LE.LBCOUNT) THEN
                  WRITE (25,113) ZLB(I,1),ZLB(I,2)
```

```
113             FORMAT (12X,1X,12X,1X,F12.6,1X,F12.6)
              ELSE
                 WRITE (25,114) ZUB(I,1),ZUB(I,2)
114              FORMAT (F12.6,1X,F12.6)
              END IF
           END IF
89      CONTINUE
        CLOSE 25
30      RETURN
        END
```

```
      SUBROUTINE SORT(ZZ,Z,COUNT,AX)
C ***************************************************************************
C *                                                                         *
C * PROCRAM NAME: SUBROUTINE SORT                                           *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 23, 1987                                                    *
C *                                                                         *
C * PURPOSE: SUBROUTINE SORT DOES A BINARY SORT OF THE ARRAY Z.             *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * USACE: CALL SORT(ZZ,Z,COUNT,AX)                                         *
C *                                                                         *
C * DISCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARCUMENTS:                                                    *
C *             AX - INTECER - THE NUMBER OF Z VALUES PLUS ONE.             *
C *             ZZ - REAL - THE Z VALUE THAT NEEDS TO BE INSERTED INTO      *
C *                  ARRAY Z.                                               *
C *     OUTPUT ARCUMENTS:                                                   *
C *             COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE             *
C *                  VALUES CONTAINED IN ARRAY Z.                           *
C *             Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT                *
C *                  PROBABILITY DISTRIBUTION FUNCTION.                     *
C *             Z(COUNT,2) - REAL - THE NUMBER OF TIMES THE VALUE OF        *
C *                  Z(COUNT,1) HAS BEEN REPEATED DURING THE                *
C *                  SIMULATION.                                            *
C *                                                                         *
C ***************************************************************************
C
      DIMENSION Z(0:40000,2)
      REAL ZZ,Z
      INTECER COUNT,AX
      Z(0,1)=-1.0E30
            IF (ZZ.GT.Z(AX-1,1).OR.AX.LE.1) THEN
               Z(AX,1)=ZZ
               Z(AX,2)=1
               COUNT=COUNT+1
               RETURN
            END IF
C
C -------------------------------------------------------------------
C
C                    START BINARY SORT
C
C -------------------------------------------------------------------
C
```

```
                IXMIN=1
                IXMAX=AX-1
40              JUMP=NINT((IXMAX-IXMIN)/2.0)+IXMIN
                DIFFER=ABS(ZZ-Z(JUMP,1))
                IF (ZZ.EQ.Z(JUMP,1).OR.DIFFER.LE.1.0E-7) THEN
                   Z(JUMP,2)=Z(JUMP,2)+1
                   AX=AX-1
                   RETURN
                END IF
                IF (ZZ.LT.Z(JUMP,1).AND.ZZ.GT.Z(JUMP-1,1)) THEN
                   DO 30 K=AX,JUMP+1,-1
                      Z(K,1)=Z(K-1,1)
                      Z(K,2)=Z(K-1,2)
30                 CONTINUE
                   Z(JUMP,1)=ZZ
                   Z(JUMP,2)=1
                   COUNT=COUNT+1
                ELSE
                   IF (ZZ.LT.Z(JUMP,1)) THEN
                      IXMAX=JUMP-1
                   ELSE
                      IXMIN=JUMP
                   END IF
                   GOTO 40
                END IF
                RETURN
                END
```

```
      SUBROUTINE SPECIAL
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE SPECIAL                                        *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 4, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE SPECIAL READS A DISTRIBUTION FUNCTION STORED IN     *
C *          A DATA FILE.  SPECIAL THEN FINDS THE NCELL LOCATION ON THE     *
C *          X AXIS GIVEN A PROBABILITY (AREA) FOR THE DISTRIBUTION         *
C *          FUNCTION.                                                      *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINE CELLR                  *
C *                                                                         *
C * USAGE: CALL SPECIAL                                                     *
C *                                                                         *
C * DISCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             NAME - CHARACTER*30 - NAME IS THE INPUT DATA FILE NAME      *
C *                    OF THE SPECIAL DISTRIBUTION FUNCTION FOR             *
C *                    RANDOM VARIABLE NUM.                                 *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                    LOCATIONS THAT APPROXIMATE THE PROBABILITY           *
C *                    DENSITY FUNCTION OF RANDOM VARIABLE NUM.             *
C *             NUM - INTEGER - VARIABLE NUM IS THE NUMBER OF THE           *
C *                    RANDOM VARIABLE.                                     *
C *     OUTPUT ARGUMENTS:                                                   *
C *             A(N) - REAL - WHERE N=NCELLS*2, ODD VALUES OF N             *
C *                    REPRESENT THE CENTER OF GRAVITY LOCATIONS AND        *
C *                    THE EVEN VALUES OF N INDICATE THE CELL               *
C *                    BOUNDARIES.                                          *
C *             B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF        *
C *                   GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILITY        *
C *                   DENSITY FUNCTION.                                     *
C *             MEAN(NUM) - REAL - MEAN(NUM) IS THE MEAN VALUE OF           *
C *                    RANDOM VARIABLE NUM.                                 *
C *             VARIANCE(NUM) - REAL - VARIANCE(NUM) IS THE VARIANCE OF     *
C *                    RANDOM VARIABLE NUM.                                 *
C *                                                                         *
C ***************************************************************************
C
      REAL A,TOTAL,DS,AREA,KAREA,MEAN,VARIANCE,B,Z,UB,LB,P
      INTEGER NUM,INC,NCELLS,COUNT,DISTR,NUMBER,UBCOUNT,LBCOUNT
      INTEGER NC1,NC2
      CHARACTER NAME*30,DISREP*10,ANSB*1
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
```

```
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CSPECIAL/DS(40000,3)
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      WRITE (3,*)
      WRITE (3,10) NUM
10    FORMAT (2X,'SPECIAL DISTRIBUTION FOR VARIABLE',I3)
      WRITE (3,20)
20    FORMAT (2X,'ENTER DATA DISTRIBUTION FILE NAME')
      READ (0,30) NAME
30    FORMAT (A30)
      WRITE (3,110) NUM
110   FORMAT (2X,'ENTER NUMBER OF CELLS FOR VARIABLE',I3)
      READ (0,*) NCELLS(NUM)
      OPEN (UNIT=26,FILE=NAME)
      READ (26,120) NC1,NC2
120   FORMAT (I5,I5)
      READ (26,40) NUMBER
40    FORMAT (I5)
      TOTAL=0.0
      DO 50 I=1,NUMBER
         READ (26,60) DS(I,1),DS(I,2),DS(I,3)
60       FORMAT (F20.7,F12.7,F12.7)
         TOTAL=TOTAL+DS(I,1)
50    CONTINUE
      MEAN(NUM)=TOTAL/FLOAT(NUMBER)
      VARIANCE(NUM)=0.0
      DO 90 I=1,NUMBER
         P=DS(I,3)/FLOAT(NC1*NC2)
         VARIANCE(NUM)=((DS(I,1)-MEAN(NUM))**2)*P+VARIANCE(NUM)
90    CONTINUE
      CLOSE 26
      INC=0
      AREA=1/(FLOAT(NCELLS(NUM))*2.0)
      DO 70 K=1,NCELLS(NUM)*2
         KAREA=AREA*(FLOAT(K))
100      INC=INC+1
         IF (DS(INC,2).GE.KAREA) THEN
            A(K)=((KAREA-DS(INC-1,2))*(DS(INC,1)-DS(INC-1,1)))/
     *           (DS(INC,2)-DS(INC-1,2))+DS(INC-1,1)
            INC=INC-1
         ELSE
            GOTO 100
         END IF
70    CONTINUE
      CALL CELLR
      RETURN
      END
```

```
      SUBROUTINE TRIANGLE
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE TRIANGLE                                       *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 4, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE TRIANGLE FINDS THE NCELL LOCATION ON THE X AXIS     *
C *          GIVEN A PROBABILITY (AREA) FOR A TRIANGULAR RANDOM             *
C *          VARIABLE.                                                      *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINE CELLR                  *
C *                                                                         *
C * USAGE: CALL TRIANGLE                                                    *
C *                                                                         *
C * DISCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             AA - REAL - VARIABLE AA IS THE MINIMUM VALUE OF THE         *
C *                  TRIANGULAR DENSITY FUNCTION.                           *
C *             BB - REAL - VARIABLE BB IS THE MAXIMUM VALUE OF THE         *
C *                  TRIANGULAR DENSITY FUNCTION.                           *
C *             CC - REAL - VARIABLE CC IS THE MEDIAN VALUE OF THE          *
C *                  TRIANGULAR DENSITY FUNCTION.                           *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                  LOCATIONS THAT APPROXIMATE THE PROBABILITY             *
C *                  THE PROBABILITY DENSITY FUNCTION OF RANDOM             *
C *                  VARIABLE NUM.                                          *
C *             NUM - INTEGER - VARIABLE NUM IS THE NUMBER OF THE           *
C *                  RANDOM VARIABLE.                                       *
C *     OUPUT ARGUMENTS:                                                    *
C *             A(N) - REAL - WHERE N=NCELLS*2, ODD VALUES OF N             *
C *                  REPRESENT THE CENTER OF GRAVITY LOCATIONS AND          *
C *                  THE EVEN VALUES OF N INDICATE THE CELL                 *
C *                  BOUNDARIES.                                            *
C *             B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF        *
C *                  GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILITY         *
C *                  DENSITY FUNCTION.                                      *
C *             MEAN(NUM) - REAL - MEAN(NUM) IS THE MEAN VALUE OF           *
C *                  RANDOM VARIABLE NUM.                                   *
C *             VARIANCE(NUM) - REAL - VARIANCE(NUM) IS THE VARIANCE OF     *
C *                  RANDOM VARIABLE NUM.                                   *
C *                                                                         *
C ***************************************************************************
C
      REAL AREA,KAREA,A,B,MEAN,VARIANCE,AA,BB,CC,TAREA,Z
      REAL ZMEAN,ZVARIANCE,UB,LB
```

```
      INTEGER NCELLS,NUM,K,COUNT,DISTR,UBCOUNT,LBCOUNT
      CHARACTER DISREP*10,ANSB
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
5     WRITE (3,*)
      WRITE (3,*)
      WRITE (3,10) NUM
10    FORMAT (2X,'TRIANGULAR DISTRIBUTION FOR VARIABLE',I3)
      WRITE (3,20)
20    FORMAT (2X,'PARAMETERS: A<C<B')
      WRITE (3,30)
30    FORMAT (2X,'RANGE: (A,B)')
      WRITE (3,40)
40    FORMAT (2X,'ENTER "A" (MINIMUM) VALUE')
      READ (0,*) AA
      WRITE (3,50)
50    FORMAT (2X,'ENTER "C" (MIDDLE) VALUE')
      READ (0,*) CC
      IF (CC-AA.LT.0) THEN
         GOTO 5
      END IF
      WRITE (3,60)
60    FORMAT (2X,'ENTER "B" (MAXIMUM) VALUE')
      READ (0,*) BB
      IF (BB-CC.LT.0) THEN
         GOTO 5
      END IF
      WRITE (3,70) NUM
70    FORMAT (2X,'ENTER NUMBER OF CELLS FOR VARIABLE',I3)
      READ (0,*) NCELLS(NUM)
      AREA=1/(FLOAT(NCELLS(NUM))*2.0)
      TAREA=(CC-AA)*(2/(BB-AA))/2
      DO 80 K=1,NCELLS(NUM)*2.0
         KAREA=AREA*(FLOAT(K))
         IF (KAREA.GT.TAREA) THEN
            A(K)=BB-SQRT((1-KAREA)*(BB-AA)*(BB-CC))
         ELSE
            A(K)=SQRT(KAREA*(BB-AA)*(CC-AA))+AA
         END IF
80    CONTINUE
      MEAN(NUM)=(AA+BB+CC)/3
      VARIANCE(NUM)=(AA**2+BB**2+CC**2-AA*BB-AA*CC-BB*CC)/18
      CALL CELLR
      RETURN
      END
```

```
      SUBROUTINE UNIFORM
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE UNIFORM                                        *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 4, 1987                                                     *
C *                                                                         *
C * PURPOSE: SUBROUTINE UNIFORM FINDS THE NCELL LOCATION ON THE X AXIS      *
C *          GIVEN A PROBABILITY (AREA) FOR A UNIFORM RANDOM VARIABLE.      *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINE CELLR                  *
C *                                                                         *
C * USAGE: CALL UNIFORM                                                     *
C *                                                                         *
C * DISCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             MAXX - REAL - MAXX IS THE MAXIMUM POSSIBLE VALUE FOR        *
C *                    THE UNIFORM RANDOM VARIABLE.                         *
C *             MINX - REAL - MINX IS THE MINIMUM POSSIBLE VALUE FOR        *
C *                    THE UNIFORM RANDOM VARIABLE.                         *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                    LOCATIONS THAT APPROXIMATE THE PROBABILITY           *
C *                    DENSITY FUNCTION OF RANDOM VARIABLE NUM.             *
C *             NUM - INTEGER - VARIABLE 'NUM' IS THE NUMBER OF THE         *
C *                    RANDOM VARIABLE.                                     *
C *     OUTPUT ARGUMENTS:                                                   *
C *             A(N) - REAL - WHERE N=NCELLS*2, ODD VALUES OF N             *
C *                    REPRESENT THE CENTER OF GRAVITY LOCATIONS AND        *
C *                    THE EVEN VALUES OF N INDICATE THE CELL               *
C *                    BOUNDARIES.                                          *
C *             B(NCELLS,NUM) - REAL - THE DISCRETE POINT (CENTER OF        *
C *                    GRAVITY) REPRESENTATIONS FOR NUM'S PROBABILITY       *
C *                    DENSITY FUNCTION.                                    *
C *             MEAN(NUM) - REAL - MEAN(NUM) IS THE MEAN VALUE OF           *
C *                    RANDOM VARIABLE NUM.                                 *
C *             VARIANCE(NUM) - REAL - VARIANCE(NUM) IS THE VARIANCE OF     *
C *                    RANDOM VARIABLE NUM.                                 *
C *                                                                         *
C ***************************************************************************
C
      REAL AREA,KAREA,MINX,MAXX,A,B,MEAN,VARIANCE,Z,ZMEAN,ZVARIANCE
      REAL UB,LB
      INTEGER NCELLS,NUM,K,COUNT,DISTR,UBCOUNT,LBCOUNT
      CHARACTER DISREP*10,ANSB*1
      COMMON/FIRST/A(0:400),B(0:400,2),Z(0:40000,3),COUNT
      COMMON/SECOND/DISTR(2),NCELLS(2),VARIANCE(2),MEAN(2),DISREP(2)
```

```
      COMMON/THIRD/NUM,ZMEAN,ZVARIANCE,UB(0:400,2),LB(0:400,2),ANSB
      COMMON/CBOUND/ZLB(0:40000,3),ZUB(0:40000,3),UBCOUNT,LBCOUNT
      WRITE (3,*)
      WRITE (3,*)
5     WRITE (3,10) NUM
10    FORMAT (2X,'UNIFORM DISTRIBUTION FOR VARIABLE',I3)
      WRITE (3,30)
30    FORMAT (2X,'RANGE (MINX,MAXX)')
      WRITE (3,40)
40    FORMAT (2X,'ENTER MINX VALUE')
      READ (0,*) MINX
      WRITE (3,50)
50    FORMAT (2X,'ENTER MAXX VALUE')
      READ (0,*) MAXX
      IF (MAXX-MINX.LT.0) THEN
         GOTO 5
      END IF
      WRITE (3,60) NUM
60    FORMAT (2X,'ENTER NUMBER OF CELLS FOR VARIABLE',I3)
      READ (0,*) NCELLS(NUM)
      AREA=1/(FLOAT(NCELLS(NUM))*2.0)
      DO 20 K=1,NCELLS(NUM)*2.0
         KAREA=AREA*(FLOAT(K))
         A(K)=MINX+KAREA*(MAXX-MINX)
20    CONTINUE
      MEAN(NUM)=(MINX+MAXX)/2.0
      VARIANCE(NUM)=((MAXX-MINX)**2)/12.0
      CALL CELLR
      RETURN
      END
```

APPENDIX B

DENSITY FUNCTION PROGRAM

```
C ************************************************************************
C *                                                                      *
C * PROGRAM NAME: DENSITY                                                *
C *                                                                      *
C * WRITTEN BY: BRUCE SWANSON                                            *
C *                                                                      *
C * DATE: MARCH 26, 1987                                                 *
C *                                                                      *
C * PURPOSE: PROGRAM "DENSITY" APPROXIMATES THE JOINT PROBABILITY        *
C *          DENSITY FUNCTION USING A CUBIC SPLINE AND A HISTOGRAM.      *
C *          THE DISTRIBUTION DATA IS READ IN FROM A FILE DETERMINED     *
C *          BY THE USER.  IF THE PROGRAM FAILS DURING THE               *
C *          CUBIC SPLINE APPROXIMATION, THE "MANAGE" PROGRAM MUST BE    *
C *          USED TO ADJUST THE DATA.                                    *
C *                                                                      *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINES HISTOGRAM          *
C *                                                    OUTPUTD           *
C *                                                    SMOOTH            *
C *                                                                      *
C * DESCRIPTION OF PARAMETERS:                                           *
C *     INPUT ARGUMENTS:                                                 *
C *             COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z        *
C *                     LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION  *
C *                     FUNCTION.                                        *
C *             NAME - CHARACTER*30 - NAME IS THE INPUT/OUTPUT DATA      *
C *                     FILE NAME OF THE DISTRIBUTION/DENSITY FUNCTION.  *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE           *
C *                     LOCATIONS THAT APPROXIMATE THE PROBABILTIY       *
C *                     DENSITY FUNCTION OF RANDOM VARIABLE NUM.         *
C *             Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT             *
C *                     PROBABILITY DISTRIBUTION FUNCTION.               *
C *             Z(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS      *
C *                     THAN OR EQUAL TO Z(COUNT,1).                     *
C *             Z(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF     *
C *                     TIMES Z(COUNT,1) WAS REPEATED DURING THE         *
C *                     SIMULATION.                                      *
C *                                                                      *
C *     OUTPUT ARGUMENTS:                                                *
C *             H(N,1) - REAL - THE Z LOCATION OF THE NTH CELL OF THE    *
C *                     HISTOGRAM.                                       *
C *             H(N,2) - REAL - THE FRACTION OF Z LOCATIONS WITHIN THE    *
C *                     RANGE OF H(N,1).                                 *
C *             NUMBER - REAL - THE NUMBER OF TOTAL DIVISIONS WITHIN     *
C *                     THE HISTOGRAM.                                   *
C *             RANGE - REAL - THE RANGE OF VALUES OF THE HISTOGRAM.     *
C *             R(N) - REAL - THE "SMOOTHED" VALUES OF THE Z             *
C *                     PROBABILITY DISTRIBUTION FOR VARIABLE N AFTER    *
C *                     THE CUBIC SPLINE APPROXIMATION.                  *
```

```
C *            R1(N) - REAL - THE DENSITY FUNCTION AFTER THE CUBIC     *
C *                    SPINE APPROXIMATION.                            *
C *            SIZE - REAL - THE SIZE OF EACH CELL WITHIN THE          *
C *                    HISTOGRAM.                                      *
C *                                                                    *
C ********************************************************************
C
C
      INTEGER COUNT,NCELLS
      REAL Z
      CHARACTER NAME*30
      COMMON/MAIN/Z(20000,4),COUNT,NCELLS(2),H(200,2),R(20004),
     *           R1(20004),NUMBER
      WRITE (3,10)
10    FORMAT (2X,'ENTER DISTRIBUTION DATA FILE NAME')
      READ (0,20) NAME
20    FORMAT (A30)
      OPEN (UNIT=23,FILE=NAME)
      READ (23,60) NCELLS(1),NCELLS(2)
60    FORMAT (I5,I5)
      READ (23,30) COUNT
30    FORMAT (I5)
      DO 40 I=1,COUNT
         READ (23,50) Z(I,1),Z(I,2),Z(I,3)
50       FORMAT (F20.7,F12.7,F12.7)
40    CONTINUE
      CLOSE 23
      CALL HISTOGRAM
      CALL SMOOTH
      CALL OUTPUTD
      STOP
      END
```

```
      SUBROUTINE HISTOGRAM
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE HISTOGRAM                                      *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 26, 1987                                                    *
C *                                                                         *
C * PURPOSE: SUBROUTINE "HISTOGRAM" BUILDS A HISTOGRAM FROM A               *
C *          DISTRIBUTION FUNCTION STORED IN A DATA FILE.                   *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * USAGE: CALL HISTOGRAM                                                   *
C *                                                                         *
C * DESCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z           *
C *                     LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION     *
C *                     FUNCTION.                                           *
C *             NAME - CHARACTER*30 - NAME IS THE OUTPUT DATA FILE NAME     *
C *                     OF THE HISTOGRAM.                                   *
C *             NCELLS(NUM) - INTEGER - THE NUMBER OF DISCRETE              *
C *                     LOCATIONS THAT APPROXIMATE THE PROBABILTIY          *
C *                     DENSITY FUNCTION OF RANDOM VARIABLE NUM.            *
C *             Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT                *
C *                     PROBABILITY DISTRIBUTION FUNCTION.                  *
C *             Z(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS         *
C *                     THAN OR EQUAL TO Z(COUNT,1).                        *
C *             Z(COUNT,3) - REAL - THE NUMBER OF TIMES THE VALUE OF        *
C *                     TIMES Z(COUNT,1) WAS REPEATED DURING THE            *
C *                     SIMULATION.                                         *
C *                                                                         *
C *     OUTPUT ARGUMENTS:                                                   *
C *             H(N,1) - REAL - THE Z LOCATION OF THE NTH CELL OF THE       *
C *                     HISTOGRAM.                                          *
C *             H(N,2) - REAL - THE FRACTION OF Z LOCATIONS WITHIN THE      *
C *                     RANGE OF H(N,1).                                    *
C *             NUMBER - REAL - THE NUMBER OF TOTAL DIVISIONS WITHIN        *
C *                     THE HISTOGRAM.                                      *
C *             RANGE - REAL - THE RANGE OF VALUES OF THE HISTOGRAM.        *
C *             SIZE - REAL - THE SIZE OF EACH CELL WITHIN THE              *
C *                     HISTOGRAM.                                          *
C *                                                                         *
C ***************************************************************************
C
      REAL RANGE,SIZE,H,Z
```

```
      INTEGER NUMBER,COUNT,NCELLS
      CHARACTER NAME*30
      COMMON/MAIN/Z(20000,4),COUNT,NCELLS(2),H(200,2),R(20004),
     *            R1(20004),NUMBER
C     COMMON/
      NUMBER=NINT(1+3.3*LOG10(FLOAT(NCELLS(1)*NCELLS(2))))
      RANGE=Z(COUNT,1)-Z(1,1)
      SIZE=RANGE/FLOAT(NUMBER)
      DO 10 I=1,NUMBER
         H(I,1)=SIZE*FLOAT(I)+Z(1,1)
         H(I,2)=0.0
10    CONTINUE
      J=1
      DO 20 I=1,COUNT
30       DIFFER=ABS(Z(I,1)-H(J,1))
         IF (Z(I,1).LE.H(J,1).OR.DIFFER.LE.1.0E-6) THEN
            H(J,2)=H(J,2)+Z(I,3)
         ELSE
            J=J+1
            GOTO 30
         END IF
20    CONTINUE
      DO 40 I=1,NUMBER
         H(I,2)=H(I,2)/(NCELLS(1)*NCELLS(2))
40    CONTINUE
      WRITE (3,50)
50    FORMAT (2X,'ENTER HISTOGRAM DATA FILE NAME')
      READ (0,60) NAME
60    FORMAT (A30)
      OPEN (UNIT=23,FILE=NAME)
      WRITE (23,70) NUMBER
70    FORMAT (I5)
      DO 90 I=1,NUMBER
         WRITE (23,80) H(I,1),H(I,2)
80       FORMAT (F12.7,F12.7)
90    CONTINUE
      CLOSE 23
      RETURN
      END
```

```
      SUBROUTINE OUTPUTD
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE OUTPUTD                                        *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 26, 1987                                                    *
C *                                                                         *
C * PURPOSE: SUBROUTINE OUTPUTD PRINTS THE JOINT PROBABILITY DENSITY        *
C *          FUNCTION TO A FILE DESIGNATED BY THE USER.                     *
C *                                                                         *
C * SUBROUTINES AND SUBPROCRAMS REQUIRED: NONE                              *
C *                                                                         *
C * USAGE: CALL OUTPUTD                                                     *
C *                                                                         *
C * DESCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             COUNT - INTEGER - COUNT IS THE NUMBER OF UNIQUE Z           *
C *                     LOCATIONS OF THE JOINT PROBABILITY DISTRIBUTION     *
C *                     FUNCTION.                                           *
C *             NAME - CHARACTER*30 - NAME IS THE OUTPUT DATA FILE NAME     *
C *                     OF THE APPROXIMATED JOINT PROBABILITY DENSTIY       *
C *                     FUNCTION.                                           *
C *             Z(COUNT,1) - REAL - THE Z VALUE OF THE JOINT                *
C *                     PROBABILITY DISTRIBUTION FUNCTION.                  *
C *             R(N) - REAL - THE "SMOOTHED" VALUES OF THE Z                *
C *                     PROBABILITY DISTRIBUTION FOR VARIABLE N AFTER       *
C *                     THE CUBIC SPLINE APPROXIMATION.                     *
C *             R1(N) - REAL - THE DENSITY FUNCTION AFTER THE CUBIC         *
C *                     SPINE APPROXIMATION.                                *
C *                                                                         *
C ***************************************************************************
C
      INTEGER NCELLS,COUNT,NUMBER
      REAL Z,RANGE,SIZE,H,R,R1
      CHARACTER NAME*30
      COMMON/MAIN/Z(20000,4),COUNT,NCELLS(2),H(200,2),R(20004),
     *            R1(20004),NUMBER
      WRITE (3,60)
60    FORMAT (2X,'ENTER SPLINE DATA FILE NAME')
      READ (0,70) NAME
70    FORMAT (A30)
      OPEN (UNIT=23,FILE=NAME)
      WRITE (23,80) COUNT
80    FORMAT (I5)
      DO 90 I=1,COUNT
         WRITE (23,100) Z(I,1),R(I),R1(I)
100      FORMAT (F20.7,F12.7,F12.7)
90    CONTINUE
      CLOSE 23
      RETURN
      END
```

```
      SUBROUTINE SMOOTH
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE SMOOTH                                         *
C *                                                                         *
C * PURPOSE: SUBROUTINE SMOOTH APPROXIMATES THE DENSITY FUNCTION FROM       *
C *          A GIVEN SET OF DISTRIBUTION DATA.                              *
C *                                                                         *
C ***************************************************************************
C
C
C     SANDIA MATHEMATICAL PROGRAM LIBRARY
C     APPLIED MATHEMATICS DIVISION 2646
C     SANDIA LABORATORIES
C     ALBUQUERQUE, NEW MEXICO  87185
C     CONTROL DATA 6600/7600  VERSION 8.1  AUCUST 1980
C                  *************************
C                  *       ISSUED BY       *
C                  *  SANDIA LABORATORIES, *
C                  *   A PRIME CONTRACTOR  *
C                  ********     TO THE     *
C                         *  UNITED STATES *
C                         *   DEPARTMENT   *
C                         *       OF       *
C                         *     ENERGY     *
C     ********************  ---NOTICE---  ********************
C     *THIS REPORT WAS PREPARED AS AN ACCOUNT OF WORK SPONSORED*
C     *  BY THE UNITED STATES GOVERNMENT.  NEITHER THE UNITED  *
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C     *          **********    ACCURACY,   **********          *
C     *          *        *  COMPLETENESS  *        *          *
C     *          *        *  OR USEFULNESS *        *          *
C     *          *        *     OF ANY     *        *          *
C     *          *        *  INFORMATION,  *        *          *
C     *          *        *   APPARATUS,   *        *          *
C     *       ****        *     PRODUCT    *        ****       *
C     *       *           *   OR PROCESS   *           *       *
C     *       *           *   DISCLOSED,   *           *       *
C     *       *           *  OR REPRESENTS *           *       *
C     *       *          **    THAT ITS    **          *       *
C     *       *          **  USE WOULD NOT **          *       *
C     *********          **    INFRINGE    **          *********
C                        **    PRIVATELY   **
C                        **      OWNED     **
C                        **     RIGHTS.    **
C                        **                **
C                        **                **
C                        **                **
C                        ********************
C
```

```
C     CONVERSION FROM THE ALGOL BY RONDALL E JONES
C     REFERENCE -- NUMERISHE MATHEMATIK 10,177-183 (1967) C H REINSCH
C
C     ABSTRACT
C        SMOO FITS A SMOOTH SPLINE THROUGH A CIVEN SET OF DATA POINTS
C        BY MINIMIZINC THE INTEGRAL OF THE SECOND DERIVATIVE SQUARED,
C        SUBJECT TO THE CONSTRAINT THAT
C                N
C              SUM ( (R(I)-Y(I))/DY(I) )**2  .LE. S
C              I=1
C        (WHERE R(I) IS THE ORDINATE OF THE SMOOTH SPLINE AT X(I).)
C        SMOO RETURNS THE VALUES OF THE SPLINE FUNCTION, R,
C        ITS FIRST DERIVATIVE, R1, AND ITS SECOND DERIVATIVE, R2,
C        EVALUATED AT THE ABSCISSAS OF THE CIVEN DATA POINTS.
C        THE RESULTING SPLINE, DEFINED BY THE ARRAYS X, R, AND R2,
C        MAY THEN BE INTERPOLATED (IF DESIRED) USING SPLINT.
C        FOR AN EXACT SPLINE FIT SEE SUBROUTINE SPLIFT.
C
C     DESCRIPTION OF ARGUMENTS
C        INPUT ARCUMENTS --
C        N  - NUMBER OF DATA VALUES (AT LEAST 3)
C        X  - ABSCISSA ARRAY (INCREASINC ORDER)
C        Y  - ORDINATE ARRAY
C        DY - ARRAY OF ERROR ESTIMATES.  DY(I) SHOULD BE AN ESTIMATE
C             OF THE ERROR (ACTUALLY, THE STANDARD DEVIATION) IN Y(I).
C             THUS, THE UNITS OF DY ARE THE SAME AS THE UNITS OF Y.
C             LARGER VALUES OF DY(I) ALLOW A LOOSER, SMOOTHER FIT.
C             SMALLER VALUES OF DY(I) CAUSE A TICHTER FIT.  SETTINC
C             DY(I)=0 AT ALL POINTS RESULTS IN AN EXACT FIT.(SEE SPLIFT)
C             BY APPROPRIATELY ADJUSTINC DY(I) AT EACH POINT, THE SPLINE
C             CAN BE MADE TICHT AT CRITICAL POINTS AND LOOSE AT OTHERS.
C        S  - SHOULD NORMALLY = N.  (NOTE-- S IS FLOATINC POINT - DONT
C             USE N DIRECTLY FOR S.)  IF YOU WISH TO TIGHTEN OR LOOSEN
C             THE SPLINE FIT BY MULTIPLYINC EACH ELEMENT OF DY BY
C             SOME FACTOR F, YOU MAY ALTERNATIVELY SIMPLY MULTIPLY
C             S BY F**2.
C        OUTPUT ARGUMENTS --
C        A,B,C,D - CUBIC BETWEEN X(I) AND X(I+1) IS
C             A(I) + B(I)*H + C(I)*H**2 + D(I)*H**3
C             WHERE H IS DESIRED ABSCISSA MINUS X(I).
C        R  - ARRAY OF SMOOTH SPLINE VALUES
C        R1 - ARRAY OF SMOOTH SPLINE DERIVATIVES
C        R2 - ARRAY OF SMOOTH SPLINE SECOND DERIVATIVES
C        T,T1,U,V - WORK ARRAYS
C        IERR- A STATUS CODE
C             --NORMAL CODE
C               =1 MEANS THE REQUESTED SPLINE WAS COMPUTED.
C             --ABNORMAL CODE
C               =2 MEANS EITHER N IS LESS THAN 3, OR S IS NECATIVE,
C                  OR THE X-AXIS VALUES ARE MISORDERED.
C
C       X,Y,DY,A,B,C,D MUST BE DIMENSIONED AT LEAST N
C       R,R1,R2,T,T1,U,V MUST BE DIMENSION AT LEAST N+2
C
```

```
C     THE ORIGINAL N1 WAS FIXED AT 1 TO AVOID WASTED WORK ARRAY SPACE
C     THE ORIGINAL N2 IS CALLED N HERE
C     ALL WORK ARRAY INDICES ARE 1 LARCER THAN IN THE ALGOL,
C     TO AVOID A ZERO SUBSCRIPT.
C
        COMMON/MAIN/Z(20000,4),COUNT,NCELLS(2),HHH(200,2),R(20004),
     *              R1(20004),NUMBER
        COMMON/SMOO/T(20004),T1(20004),U(20004),V(20004),R2(20004),
     *X(20000),Y(20000),DY(20000),A(20000),B(20000),C(20000),D(20000)
        INTECER COUNT
        REAL AAA,BBB,Z,R2,X,Y,DY,A,B,C,D
        WRITE (3,66)
66      FORMAT(2X,'ENTER TOLERANCE FOR SPLINE FIT')
        READ (0,*) F
        S=.05**2
C       S=FLOAT(COUNT)
        N=COUNT
        DO 15 I=1,COUNT
           X(I)=Z(I,1)
           Y(I)=Z(I,2)
           DY(I)=F
15      CONTINUE
C
C       CHECK INPUT DATA
C
        IERR = 1
        IF (N.LT.3) GO TO 10
        IF (S.LT.0.0) GO TO 10
        DO 5 I=2,N
        IF (X(I)-X(I-1)) 10,10,5
     5  CONTINUE
        GO TO 20
    10  WRITE (3,76)
76      FORMAT (2X,'PROGRAM BOMBED BECAUSE OF MISSORDER')
        IERR = 2
        RETURN
    20  CONTINUE
C
C       INITIALIZE
C
        R(1) = 0.0
        R(2) = 0.0
        R1(N+1) = 0.0
        R2(N+1) = 0.0
        R2(N+2) = 0.0
        U(1) = 0.0
        U(2) = 0.0
        U(N+1) = 0.0
        U(N+2) = 0.0
        P = 0.0
        M2 = N-1
C
        H = X(2)-X(1)
        IF (H.EQ.0.0) THEN
```

```
        WRITE (3,439) X(2),X(1)
439     FORMAT (2X,'X(2)=',F12.7,'X(1)=',F12.7)
      END IF
      F = (Y(2)-Y(1))/H
      DO 100 I=2,M2
      G = H
      H = X(I+1)-X(I)
      E = F
      F = (Y(I+1)-Y(I))/H
      A(I) = F-E
      T(I+1) = 2.0*(G+H)/3.0
      T1(I+1) = H/3.0
      R2(I+1) = DY(I-1)/G
      R(I+1) = DY(I+1)/H
  100 R1(I+1) = -DY(I)/G - DY(I)/H
C
      DO 200 I=2,M2
      B(I)= R(I+1)*R(I+1) + R1(I+1)*R1(I+1) + R2(I+1)*R2(I+1)
      C(I)= R(I+1)*R1(I+2) + R1(I+1)*R2(I+2)
  200 D(I)= R(I+1)*R2(I+3)
      F2  = -S
C
C     NEXT ITERATION
C
  250 DO 300 I=2,M2
      R1(I) = F*R(I)
      R2(I-1) = G*R(I-1)
      R(I+1)=1.0/(P*B(I)+T(I+1)-F*R1(I)-G*R2(I-1))
      U(I+1)=A(I)-R1(I)*U(I)-R2(I-1)*U(I-1)
      F = P*C(I)+T1(I+1)-H*R1(I)
      G = H
  300 H = D(I)*P
C
      DO 400 J=2,M2
      I=M2-(J-2)
  400 U(I+1)=R(I+1)*U(I+1)-R1(I+1)*U(I+2)-R2(I+1)*U(I+3)
      E=0.0
      H=0.0
C
      DO 500 I=1,M2
      G = H
      H = (U(I+2)-U(I+1))/(X(I+1)-X(I))
      V(I+1) = (H-G)*DY(I)*DY(I)
  500 E = E+V(I+1)*(H-G)
      V(N+1) = -H*DY(N)*DY(N)
      G = V(N+1)
      E = E-G*H
      G = F2
      F2= E*P*P
      IF ((F2.GE.S).OR.(F2.LE.G)) GO TO 650
      F = 0.0
      H = (V(3)-V(2))/(X(2)-X(1))
C
      DO 600 I=2,M2
```

```
      G = H
      H = (V(I+2)-V(I+1))/(X(I+1)-X(I))
      G=H-G-R1(I)*R(I)-R2(I-1)*R(I-1)
      F = F+G*R(I+1)*G
  600 R(I+1) = G
      H = E-P*F
      IF (H.LE.0.0) GO TO 650
      P = P+(S-F2)/((SQRT(S/E)+P)*H)
      GO TO 250
C
C     FINISH
C
  650 DO 700 I=1,N
      A(I)=Y(I)-P*V(I+1)
  700 C(I)=U(I+1)
C
      DO 800 I=1,M2
      H=X(I+1)-X(I)
      D(I)=(C(I+1)-C(I))/(3.0*H)
  800 B(I)=(A(I+1)-A(I))/H-(H*D(I)+C(I))*H
      B(N)=0.0
      D(N)=0.0
C
C     OUTPUT DERIVATIVES, ETC.
C
      DO 900 I=1,M2
      R(I) = A(I)
      R1(I) = B(I)
  900 R2(I) = 2.0*C(I)
      R(N) = A(N)
      H = X(N)-X(M2)
      R1(N)=(3.0*D(M2)*H+2.0*C(M2))*H+B(M2)
      R2(N) = 0.0
      DO 16 I=1,COUNT
         Z(I,3)=R(I)
         Z(I,4)=R1(I)
16    CONTINUE
      RETURN
      END
```

APPENDIX C

DATA MANAGEMENT PROGRAM

```
C ************************************************************************
C *                                                                      *
C * PROGRAM NAME: MANAGE                                                 *
C *                                                                      *
C * WRITTEN BY: BRUCE SWANSON                                            *
C *                                                                      *
C * DATE: MARCH 6, 1987                                                  *
C *                                                                      *
C * PURPOSE: PROGRAM "MANAGE" IS USED IF A PROBLEM OCCURS WITH THE       *
C *          CALCULATION OF THE CUBIC SPLINE DENSITY APPROXIMATION       *
C *          PROGRAM.  THIS PROGRAM READS IN A DISTRIBUTION DATA FILE    *
C *          AND FINDS THE SMALLEST ABSCISSA AND PRINTS OUT THE VALUE    *
C *          TO THE SCREEN.  IT THEN ELIMINATES ANY DATA POINTS THAT     *
C *          ARE CLOSER THAN 0.000001 ALONG THE X AXIS.                  *
C *                                                                      *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                           *
C *                                                                      *
C * DESCRIPTION OF PARAMETERS:                                           *
C *     INPUT ARGUMENTS:                                                 *
C *             ANSWER - CHARACTER*1 - (Y OR N) AN INPUT VARIABLE USED   *
C *                      TO INDICATED IF THE USER WANTS TO PERFORM       *
C *                      DATA MANAGEMENT TO THE DISTRIBUTION FILE.       *
C *             COUNT - INTEGER - NUMBER OF UNIQUE Z LOCATIONS.          *
C *             NAME - CHARACTER*30 - NAME IS THE INPUT DATA FILE NAME   *
C *                      OF THE JOINT DISTRIBUTION FUNCTION.             *
C *             NAME2 - CHARACTER*30 - NAME2 IS THE OUTPUT DATA FILE     *
C *                      NAME OF THE JOINT DISTRIBUTION FUNCTION.        *
C *             NCELLS(1 OR 2) - INTEGER - THE NUMBER OF DISCRETE        *
C *                      LOCATIONS THAT APPROXIMATE THE PROBABILTIY      *
C *                      DENSITY FUNCTION OF RANDOM VARIABLE 1 OR 2.     *
C *             SMALL - REAL - THE SMALLEST X INCREMENT VALUE OF THE     *
C *                      JOINT DISTRIBUTION FUNCTION.                    *
C *             Z(COUNT,1) - REAL - THE ABSISCA VALUE OF THE JOINT       *
C *                      PROBABILITY DISTRIBUTION FUNCTION.              *
C *             Z(COUNT,2) - REAL - THE PROBABILITY OF Z BEING LESS      *
C *                      THAN OR EQUAL TO Z(COUNT,1).                    *
C *                                                                      *
C *     OUTPUT ARGUMENTS:                                                *
C *             INC - REAL - THE NUMBER OF UNIQUE Z LOCATIONS AFTER      *
C *                      THE DATA MANAGEMENT HAS BEEN PERFORMED.         *
C *             ZD(INC,1) - REAL - THE ABSISCA VALUE OF THE JOINT        *
C *                      PROBABILITY DISTRIBUTION FUNCTION AFTER DATA    *
C *                      MANAGEMENT HAS BEEN PERFORMED.                  *
C *             ZD(INC,2) - REAL - THE PROBABILITY OF Z BEING LESS THAN  *
C *                      OR EQUAL TO Z(INC,1).                           *
C *                                                                      *
C ************************************************************************
C
```

```
      REAL Z,DIFF1,DIFF2,SMALL,ZD,TOLERANCE
      INTEGER COUNT,NCELLS1,NCELLS2
      CHARACTER NAME*20,ANSWER*1,NAME2*20
      COMMON/MAIN/Z(2,10000),ZD(2,10000)
      TOLERANCE=.00001
      SMALL=99999
      WRITE (3,10)
10    FORMAT (2X,'ENTER INPUT FILE NAME')
      READ (0,20) NAME
20    FORMAT (A20)
      OPEN (UNIT=23,FILE=NAME)
      READ (23,30) NCELLS1,NCELLS2
30    FORMAT (I5,I5)
      READ (23,40) COUNT
40    FORMAT (I5)
      DO 50 I=1,COUNT
         READ (23,60) Z(1,I),Z(2,I),DUMMY
60       FORMAT (F20.7,F12.7,F12.7)
50    CONTINUE
      DO 70 I=2,COUNT
         DIFF1=ABS(Z(1,I)-Z(1,I-1))
         IF (DIFF1.LT.SMALL) THEN
            SMALL=DIFF1
         END IF
70    CONTINUE
      WRITE (3,80) SMALL
80    FORMAT (2X,'SMALLEST X INCREMENT=',F12.7)
      CLOSE 23
      WRITE (3,90)
90    FORMAT (2X,'PERFORM DATA MANAGEMENT? (Y/N)')
      READ (0,100) ANSWER
100   FORMAT (A1)
      IF (ANSWER.EQ.'N') THEN
         GOTO 9999
      END IF
      WRITE (3,200)
200   FORMAT (2X,'ENTER OUTPUT FILE NAME')
      READ (0,210) NAME2
210   FORMAT(A20)
      OPEN (UNIT=24,FILE=NAME2)
      INC=1
      ZD(1,1)=Z(1,1)
      ZD(2,1)=Z(2,1)
      DO 110 I=2,COUNT
        DIFF1=ABS(Z(1,I)-Z(1,I-1))
        IF (DIFF1.LT.TOLERANCE) THEN
           GOTO 110
        ELSE
```

```
            ZD(1,INC)=Z(1,I)
            ZD(2,INC)=Z(2,I)
            INC=INC+1
          END IF
110     CONTINUE
        INC=INC-1
        WRITE (24,220) NCELLS1,NCELLS2
220     FORMAT (I5,I5)
        WRITE (24,130) INC
130     FORMAT (I5)
        DO 120 I=1,INC
           Z(I,1)=ZD(I,1)
           Z(I,2)=ZD(I,2)
           WRITE (24,140) ZD(1,I),ZD(2,I)
140        FORMAT (F20.7,F12.7)
120     CONTINUE
        CLOSE 24
9999    STOP
        END
```

APPENDIX D

GRAPHICS PROGRAM

```
C ***********************************************************************
C *                                                                     *
C * PROGRAM NAME: GRAPH                                                 *
C *                                                                     *
C * WRITTEN BY: BRUCE SWANSON                                           *
C *                                                                     *
C * DATE: MARCH 27, 1987                                                *
C *                                                                     *
C * PURPOSE: PROGRAM "GRAPH" PLOTS THE JOINT DISTRIBUTION AND DENSITY   *
C *          FUNCTIONS ON A HIREZ SELANAR GRAPHICS VT100 TERMINAL.      *
C *                                                                     *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: SUBROUTINES AUTOSCL           *
C *                                                   GVTDENSITY        *
C *                                                   GVTDISTRIBUTION   *
C *                                                   LABEL             *
C *                                                   PLLABLE           *
C *                                                   PLOTT             *
C *                                                   TICKS             *
C *                                                                     *
C * NOTE:  SUBROUTINES AUTOSCL, LABEL, PLLABEL, PLOTT, AND TICKS WERE   *
C *          WRITTEN BY DONALD A. SMITH AND ARE USED BY PERMISSION.     *
C *                                                                     *
C *                                                                     *
C * DESCRIPTION OF PARAMETERS:                                          *
C *     INPUT ARGUMENTS:                                                *
C *             COUNT - INTEGER - THE NUMBER OF POINTS TO BE PLOTTED.   *
C *             GTITLE - CHARACTER*35 - THE TITLE OF THE GRAPH.         *
C *             XLABEL - CHARACTER*30 - THE LABEL ON THE X AXIS OF THE  *
C *                      GRAPH.                                         *
C *             XX(N,M) - REAL - THE X VALUE FOR VARIABLE M, CURVE      *
C *                       NUMBER N.                                     *
C *             YLABEL - CHARACTER*30 - THE LABEL ON THE Y AXIS OF THE  *
C *                      GRAPH.                                         *
C *             YY(N,M) - REAL - THE Y VALUE FOR VARIABLE M, CURVE      *
C *                       NUMBER N.                                     *
C *                                                                     *
C ***********************************************************************
C
      REAL MATL,MITL,XX,YY,ZZ,ERROR,MAXDEV
      INTEGER COUNT,INCELLS,ICOUNT,ITIME,ANS
      CHARACTER XLABEL*30,YLABEL*30,GTITLE*35,NAME*20
      LOGICAL AUTOSC
      COMMON/ONE/XX(6,10000),YY(6,10000),COUNT
      COMMON/MISC/INCELLS(10000),ERROR(10000),MAXDEV(10000),
     *            ICOUNT(10000),ITIME(10000)
1     WRITE (3,10)
10    FORMAT (2X,'    PLOTT:')
      WRITE (3,20)
```

```
20      FORMAT (2X,'1.  DISTRIBUTION FUNCTION')
        WRITE (3,30)
30      FORMAT (2X,'2.  DENSITY FUNCTION')
        WRITE (3,40)
40      FORMAT (2X,'ENTER NUMBER FOR DESIRED CURVE')
        READ (0,*) ANS
        IF (ANS.EQ.1) THEN
           CALL GVTDISTRIBUTION
        ELSE
           IF (ANS.EQ.2) THEN
              CALL GVTDENSITY
           ELSE
              GOTO 1
           END IF
        END IF
        ICURVES=1
        WRITE (3,*)
        DO 6000 I=1,COUNT
           WRITE (3,*) XX(1,I),YY(1,I)
6000    CONTINUE
        WRITE (3,90)
90      FORMAT (2X,'ENTER TITLE')
        READ (0,100) GTITLE
100     FORMAT (A35)
        WRITE (3,92)
92      FORMAT (2X,'ENTER X LABEL')
        READ (0,93) XLABEL
93      FORMAT (A30)
        WRITE (3,94)
94      FORMAT (2X,'ENTER Y LABEL')
        READ (0,96) YLABEL
96      FORMAT (A30)
        MATL=0.03
        MITL=0.01
        IDEV=2
        AUTOSC=.TRUE.
        CALL PLOTT (D1,D2,D3,D4,MATL,MITL,XLABEL,YLABEL,
     *GTITLE,XX,YY,COUNT,ICURVES,IDEV,AUTOSC)
        END
```

```
      SUBROUTINE AUTOSCL (XX,YY,K,XMAX,XMIN,YMAX,YMIN,MATSX,MITSX,
     @                    MATSY,MITSY,AUTOSC,XSTART,YSTART,
     @                    DX,DY)
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE AUTOSCL                                        *
C *                                                                         *
C * WRITTEN BY: DONALD A. SMITH                                             *
C *                                                                         *
C *                                                                         *
C *                                                                         *
C ***************************************************************************
C
      REAL XX(6,10000),YY(6,10000),MATSX,MATSY,MITSX,MITSY,DX(4),
     @               DY(4)
      LOGICAL AUTOSC
      IF (AUTOSC) THEN
      XMAX=XX(1,1)
      XMIN=XX(1,1)
      YMAX=YY(1,1)
      YMIN=YY(1,1)
      WRITE (3,2222)
2222  FORMAT (2X'ENTERED AUTOSCL')
      DO 10 I=1,K
          IF (XX(1,I).GT.XMAX) XMAX=XX(1,I)
          IF (XX(1,I).LT.XMIN) XMIN=XX(1,I)
          IF (YY(1,I).GT.YMAX) YMAX=YY(1,I)
          IF (YY(1,I).LT.YMIN) YMIN=YY(1,I)
 10   CONTINUE
      ENDIF
      A=ABS((XMAX-XMIN)/5.)
      MATSX=A
      C=ABS((YMAX-YMIN)/5.)
      MATSY=C
      IF ((XMAX.GT.0.) .AND. (ABS(XMAX-XMIN).LT.ABS(XMAX)))  THEN
              XSTART=XMIN
              GOTO 14
      ENDIF
      IF ((XMAX.LE.0.) .AND. (ABS(XMAX-XMIN).LT.ABS(XMIN)))  THEN
              XSTART=XMAX
            ELSE
              XSTART=0.
      ENDIF
 14   CONTINUE
      IF ((YMAX.GT.0.) .AND. (ABS(YMAX-YMIN).LT.ABS(YMAX)))  THEN
              YSTART=YMIN
              GOTO 30
       ENDIF
```

```
      IF ((YMAX.LE.0.) .AND. (ABS(XMAX-XMIN).LT.ABS(XMIN))) THEN
            YSTART=YMAX
         ELSE
            YSTART=0.
      ENDIF
   30 CONTINUE
D     WRITE (3,*) 'TICKS CALC DX'
  210 CONTINUE
      DX(1)=(XMAX-XSTART)/MATSX
      DX(2)=(XMIN-XSTART)/MATSX
      MITSX=MATSX/5.
      DX(3)=(XMAX-XSTART)/MITSX
      DX(4)=(XMIN-XSTART)/MITSX
      DO 200 I=1,4
         DX(I)=INT(DX(I))
         DX(I)=REAL(DX(I))
         DX(I)=ABS(DX(I))
         IF ((I.LT.3) .AND. (DX(I).GT.20.)) THEN
            MATSX=MATSX*1.2
            GOTO 210
         ENDIF
  200 CONTINUE
D     WRITE (3,*) 'TICKS CALC DY'
311   CONTINUE
      DY(1)=(YMAX-YSTART)/MATSY
      DY(2)=(YMIN-YSTART)/MATSY
      MITSY=MATSY/5.
      DY(3)=(YMAX-YSTART)/MITSY
      DY(4)=(YMIN-YSTART)/MITSY
      DO 300 I=1,4
          DY(I)=INT(DY(I))
          DY(I)=REAL(DY(I))
          DY(I)=ABS(DY(I))
          IF ((I.LT.3) .AND.(DY(I).GT.20.)) THEN
             MATSY=1.2*MATSY
             GOTO 311
          ENDIF
  300 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GVTDENSITY
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE GVTDENSITY                                     *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 27, 1987                                                    *
C *                                                                         *
C * PURPOSE: SUBROUTINE "GVTDENSITY" READS IN THE DATA FORMATED FOR          *
C *          THE JOINT DENSITY FUNCTION.                                    *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * DESCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             COUNT - INTEGER - THE NUMBER OF POINTS TO BE PLOTTED.       *
C *                     GRAPH.                                              *
C *             XX(N,M) - REAL - THE X VALUE FOR VARIABLE M, CURVE          *
C *                     NUMBER N, FOR THE DENSITY FUNCTION.                 *
C *             YY(N,M) - REAL - THE Y VALUE FOR VARIABLE M, CURVE          *
C *                     NUMBER N, FOR THE DENSITY FUNCTIRON.                *
C *                                                                         *
C ***************************************************************************
C
      REAL MATL,MITL,XX,YY,ZZ,ERROR,MAXDEV
      INTEGER COUNT,INCELLS,ICOUNT,ITIME
      CHARACTER XLABEL*30,YLABEL*30,GTITLE*35,NAME*20
      COMMON/ONE/XX(6,10000),YY(6,10000),COUNT
      COMMON/MISC/INCELLS(10000),ERROR(10000),MAXDEV(10000),
     *            ICOUNT(10000),ITIME(10000)
      WRITE (3,51)
51    FORMAT (2X,'PLOTTING ROUTINE FOR DENSITY FUNCTION')
      WRITE (3,50)
50    FORMAT (2X,'ENTER DATA FILE NAME')
      READ (0,60) NAME
60    FORMAT (A20)
      OPEN (UNIT=23,FILE=NAME)
      READ (23,987) COUNT
987   FORMAT (I5)
      WRITE (3,*) COUNT
      DO 30 I=1,COUNT
         READ (23,628) XX(1,I),DUMMY,YY(1,I)
628      FORMAT (F20.7,F12.7,F12.7)
30    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GVTDISTRIBUTION
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE GVTDISTRIBUTION                                *
C *                                                                         *
C * WRITTEN BY: BRUCE SWANSON                                               *
C *                                                                         *
C * DATE: MARCH 27, 1987                                                    *
C *                                                                         *
C * PURPOSE: SUBROUTINE "GVTDISTRIBUTION" READS IN THE DATA FORMATED        *
C *          FOR THE JOINT DISTRIBUTION FUNCTION.                           *
C *                                                                         *
C * SUBROUTINES AND SUBPROGRAMS REQUIRED: NONE                              *
C *                                                                         *
C * DESCRIPTION OF PARAMETERS:                                              *
C *     INPUT ARGUMENTS:                                                    *
C *             COUNT - INTEGER - THE NUMBER OF POINTS TO BE PLOTTED.       *
C *                     GRAPH.                                              *
C *             XX(N,M) - REAL - THE X VALUE FOR VARIABLE M, CURVE          *
C *                     NUMBER N, FOR THE DISTRIBUTION FUNCTION.            *
C *             YY(N,M) - REAL - THE Y VALUE FOR VARIABLE M, CURVE          *
C *                     NUMBER N, FOR THE DISTRIBUTION FUNCTION.            *
C *                                                                         *
C ***************************************************************************
C
      REAL MATL,MITL,XX,YY,ZZ,ERROR,MAXDEV
      INTEGER COUNT,INCELLS,ICOUNT,ITIME
      CHARACTER XLABEL*30,YLABEL*30,GTITLE*35,NAME*20
      COMMON/ONE/XX(6,10000),YY(6,10000),COUNT
      COMMON/MISC/INCELLS(10000),ERROR(10000),MAXDEV(10000),
     *            ICOUNT(10000),ITIME(10000)
      WRITE (3,5000)
5000  FORMAT (2X,'PLOTTING ROUTINE FOR THE DISTRIBUTION FUNCTION')
      WRITE (3,50)
50    FORMAT (2X,'ENTER DATA FILE NAME')
      READ (0,60) NAME
60    FORMAT (A20)
      OPEN (UNIT=23,FILE=NAME)
      READ (23,4001) IDUMMY1,IDUMMY2
4001  FORMAT (I5,I5)
      READ (23,987) COUNT
987   FORMAT (I5)
      WRITE (3,*) COUNT
      DO 30 I=1,COUNT
         READ (23,628) XX(1,I),YY(1,I),DUMMY
628      FORMAT (F20.7,F12.7,F12.7)
      WRITE (3,204) XX(1,I),YY(1,I)
204   FORMAT (F12.7,F12.7)
```

```
30      CONTINUE
        RETURN
        END
```

```
      SUBROUTINE LABEL (XMAX,XMIN,YMAX,YMIN,DX,DY,MATSX,MATSY,
     &                  XSTART,YSTART)
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE LABEL                                          *
C *                                                                         *
C * WRITTEN BY: DONALD A. SMITH                                             *
C *                                                                         *
C *                                                                         *
C *                                                                         *
C ***************************************************************************
C
      CHARACTER CNT*10
      REAL DX(20),DY(20),MATSX,MATSY
      XZ=ABS(XMAX-XMIN)
      YZ=ABS(YMAX-YMIN)
      CALL JCOLOR(0)
      CALL JMOVE (XMIN,YSTART)
      CALL JRMOVE(-XZ*.1,0.)
      CALL JJUST (3,2)
      CALL JFONT (1)
      A= (4./100.)*XZ
      B= (3./100.)*YZ
      CALL JSIZE (A,B)
      DO 50 I=1,2
          IF (DY(I).LT.0.9) GOTO 60
          DO 70 CTT=0.,DY(I)
            WRITE (CNT,'(F10.2)') CTT*MATSY+YSTART
            CALL JHSTRG (CNT)
            CALL JRMOVE (0.0,MATSY)
 70       CONTINUE
 60       CONTINUE
          CALL JMOVE (XMIN,YSTART)
          CALL JRMOVE (-XZ*.1,0.)
          MATSY=-MATSY
 50   CONTINUE
      CALL JBASE (0.,1.,0.)
      CALL JMOVE (XSTART,YMIN)
      CALL JRMOVE (0.,-YZ*.1)
      CALL JSIZE (B*3./2.,A*2./3.)
D     WRITE (3,*) DX(1),'DX1',DX(2),'DX2'
D     WRITE (3,*) MATSX,'MATSX',YMIN,'YMIN',YMAX,'YMAX'
      DO 10 I=1,2
          IF (DX(I).LT.0.9) GOTO 20
          DO 30 CTT=0.,DX(I)
            WRITE (CNT,'(F10.2)') CTT*MATSX+XSTART
            CALL JHSTRG (CNT)
            CALL JRMOVE (MATSX,0.0)
```

```
30        CONTINUE
20        CONTINUE
          CALL JMOVE(XSTART,YMIN)
          CALL JRMOVE(0.,-YZ*.1)
          MATSX=-MATSX
10    CONTINUE
      CALL JBASE (1.,0.,0.)
      RETURN
      END
```

```
      SUBROUTINE PLLABEL (XMAX,XMIN,YMAX,YMIN,XLABEL,YLABEL,GTITLE)
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE PLLABEL                                        *
C *                                                                         *
C * WRITTEN BY: DONALD A. SMITH                                             *
C *                                                                         *
C *                                                                         *
C *                                                                         *
C ***************************************************************************
C
      CHARACTER XLABEL*30,YLABEL*30,GTITLE*35
      CALL JPEDGE (2)
      CALL JCOLOR(0)
C     CALL JRECT (XMIN,YMIN,XMAX,YMAX)
      C=ABS(( 6./100.)*(XMAX-XMIN))
      D=ABS((5./100.)*(YMAX-YMIN))
C     CALL NOBLNK (GTITLE)
      CALL JSIZE (C,D)
      CALL JJUST (1,2)
C     CALL JMOVE (XMIN,YMAX)
      CALL JMOVE (-9.*(XMAX-XMIN)/19.,YMAX)
      CALL JRMOVE (0.,D*1.25)
      CALL JHSTRG (GTITLE)
      CALL JSIZE (0.66*C,C.66*D)
      CALL JMOVE (XMIN,YMIN)
      CALL JRMOVE (0.,-D*.4/.05)
      CALL JHSTRG (XLABEL)
      CALL JMOVE (XMIN,YMIN)
      CALL JRMOVE (-C*.4/.06,0.)
      CALL JBASE (0.,1.,0.)
      CALL JSIZE (0.66*D,0.66*C)
C     CALL JBASE (0.,1.,0.)
      CALL JHSTRG (YLABEL)
      CALL JBASE (1.,0.,0.)
      RETURN
      END
```

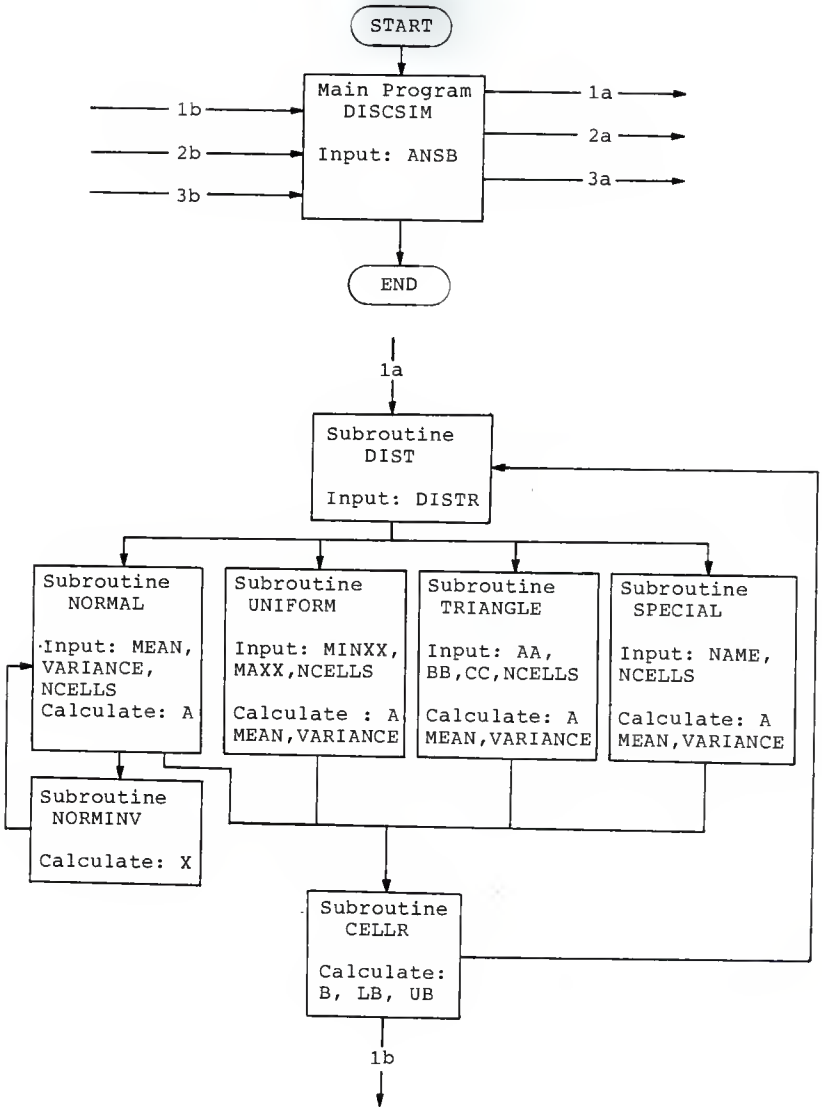
```
      SUBROUTINE PLOTT (XMAX,XMIN,YMAX,YMIN,MATL,MITL,
     @                  XLABEL,YLABEL,GTITLE,XX,YY,IDATN,
     @                  IDATNY,IDEV,AUTOSC)
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE PLOTT                                          *
C *                                                                         *
C * WRITTEN BY: DONALD A. SMITH                                             *
C *                                                                         *
C *                                                                         *
C *                                                                         *
C ***************************************************************************
C
      REAL MATL,MITL,MATSX,MITSX,MATSY,MITSY,DX(4),DY(4),
     @     XX(6,10000),YY(6,10000),MALX,MALY,MILX,MILY
      LOGICAL AUTOSC
      CHARACTER XLABEL*30,YLABEL*30,GTITLE*35
D     IF (AUTOSC) WRITE (3,*) 'AUTOSC TRUE PLOTT SUB'
        CALL AUTOSCL(XX(1,1),YY(1,1),IDATN,XMAX,XMIN,YMAX,YMIN,MATSX,
     @               MITSX,MATSY,MITSY,AUTOSC,XSTART,YSTART,DX,DY)
D     IF (AUTOSC) THEN
D       WRITE (3,*) 'JUST DID AUTO SCALE'
D     ELSE
D       WRITE (3,*) 'NO AUTO SCALE'
D     ENDIF
      CALL JBEGIN
      CALL JDINIT (IDEV)
      CALL JDEVON (IDEV)
      CALL JVPORT (-0.3,0.7,-0.2,0.8)
C     IF (ABS(XMAX-XMIN).LT..1) XMIN=XMAX-.1
C     IF (ABS(YMAX-YMIN).LT..1) YMIN=YMAX-.1
      CALL JWINDO (XMIN,XMAX,YMIN,YMAX)
      CALL JDEVWN (IDEV,-1.0,1.0,-0.7,1.0)
      CALL JOPEN
C
D     WRITE (3,*) 'OPENED WINDOW'
      IW=10000
      MALY=MATL*ABS(XMAX-XMIN)
      MALX=MATL*ABS(YMAX-YMIN)
      MILY=MITL*ABS(XMAX-XMIN)
      MILX=MITL*ABS(YMAX-YMIN)
D     WRITE (3,*) '1ST TICKS CALL'
      CALL TICKS  (XMAX,XMIN,YMAX,YMIN,MATSX,MATSY,MALX,MALY,DX(1),
     &             DY(1),IW,XSTART,YSTART)
D     WRITE (3,*) '1ST LABEL CALL'
      CALL LABEL (XMAX,XMIN,YMAX,YMIN,DX(1),DY(1),MATSX,MATSY,
     &            XSTART,YSTART)
      IW=8000
```

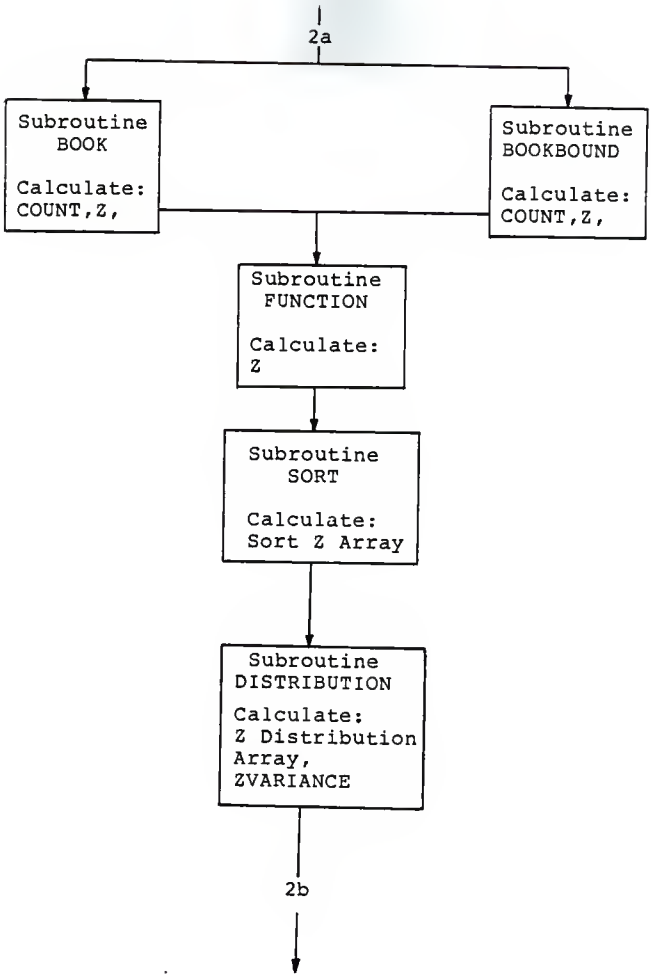
```
D     WRITE (3,*) '2ND TICKS CALL'
      CALL TICKS  (XMAX,XMIN,YMAX,YMIN,MITSX,MITSY,MILX,MILY,
     &             DX(3),DY(3),IW,XSTART,YSTART)
D     WRITE (3,*) 'PLLABEL CALL'
      CALL PLLABEL (XMAX,XMIN,YMAX,YMIN,XLABEL,YLABEL,GTITLE)
      CALL JJUST (2,2)
      RR=(6./100.)*ABS(XMAX-XMIN)
      TT=(6./100.)*ABS(YMAX-YMIN)
      CALL JSIZE (RR,TT)
      CALL JJUST (2,2)
      CALL JSIZE (RR*2./6.,TT*2./6.)
      DO 500 J=1,IDATNY
C     CALL JCOLOR(J)
      IF (J.GT.1) THEN
         CALL JLSTYL(3)
      END IF
      CALL JMOVE (XX(IDATNY,1),YY(IDATNY,1))
D     WRITE (3,*) XX(IDATNY,1),YY(IDATNY,1)
      DO 100 I=2,IDATN
      IF ((XX(J,I).LE.XMAX) .AND.
     &   (XX(J,I).GE.XMIN) .AND.
     &   (YY(J,I).LE.YMAX) .AND.
     &   (YY(J,I).GE.YMIN)) THEN
         CALL JDRAW (XX(J,I),YY(J,I))
D        WRITE(3,*) XX(J,I),YY(J,I)
D        WRITE(3,*) J,'FIELD',I,'COUNT'
      ENDIF
 100  CONTINUE
 500  CONTINUE
C
      CALL JCLOSE
      CALL JPAUSE (IDEV)
      CALL JDEVOF (IDEV)
      CALL JDEND (IDEV)
      CALL JEND
      RETURN
      END
```

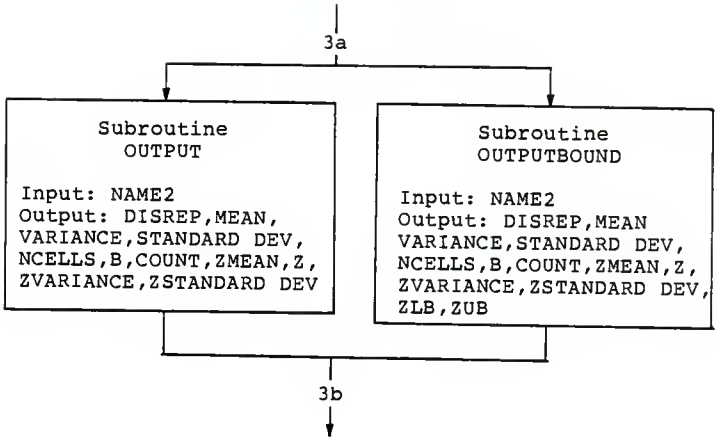
```
      SUBROUTINE TICKS (XMAX,XMIN,YMAX,YMIN,MATSX,MATSY,MLX,MLY,
     @                  DX,DY,IW,XSTART,YSTART)
C ***************************************************************************
C *                                                                         *
C * PROGRAM NAME: SUBROUTINE TICKS                                          *
C *                                                                         *
C * WRITTEN BY: DONALD A. SMITH                                             *
C *                                                                         *
C *                                                                         *
C *                                                                         *
C ***************************************************************************
C
      REAL MATSX,MATSY,MLX,MLY,DX(20),DY(20)
      CALL JLSTYL (0)
      CALL JCOLOR (7)
      CALL JLWIDE (20000)
      CALL JMOVE (XMIN,YSTART)
      CALL JDRAW (XMAX,YSTART)
      CALL JMOVE (XSTART,YMAX)
      CALL JDRAW (XSTART,YMIN)
      CALL JMOVE (XSTART,YSTART)
D     WRITE (3,*) XSTART,'XSTART ',YSTART,'YSTART'
      CALL JLWIDE (IW)
      DO 20 I=1,2
      DO 10 SP=1.,DX(I)
          CALL JRMOVE (MATSX,-MLX/2)
          CALL JRDRAW (0.,MLX)
          CALL JRMOVE (0.,-MLX/2)
 10   CONTINUE
 15   CONTINUE
      CALL JMOVE (XSTART,YSTART)
      MATSX=-MATSX
 20   CONTINUE
      CALL JMOVE (XSTART,YSTART)
D     WRITE (3,*) 'TICKS CALC DY'
      DO 30 I=1,2
      DO 40 SP=1.,DY(I)
          CALL JRMOVE (-MLY/2.,MATSY)
          CALL JRDRAW (MLY,0.)
          CALL JRMOVE (-MLY/2.,0.)
 40   CONTINUE
 45   CONTINUE
      CALL JMOVE (XSTART,YSTART)
      MATSY=-MATSY
 30   CONTINUE
      RETURN
      END
```

APPENDIX E

FLOWCHART FOR THE DISCSIM PROGRAM


START
Main Program
DISCSIM
Input: ANSB
1b
2b
3b
1a
2a
3a
END
1a
Subroutine
DIST
Input: DISTR
Subroutine
NORMAL
Input: MEAN,
VARIANCE,
NCELLS
Calculate: A
Subroutine
UNIFORM
Input: MINXX,
MAXX,NCELLS
Calculate : A
MEAN,VARIANCE
Subroutine
TRIANGLE
Input: AA,
BB,CC,NCELLS
Calculate: A
MEAN,VARIANCE
Subroutine
SPECIAL
Input: NAME,
NCELLS
Calculate: A
MEAN,VARIANCE
Subroutine
NORMINV
Calculate: X
Subroutine
CELLR
Calculate:
B, LB, UB
1b


2a
Subroutine
BOOK
Calculate:
COUNT,Z,
Subroutine
BOOKBOUND
Calculate:
COUNT,Z,
Subroutine
FUNCTION
Calculate:
Z
Subroutine
SORT
Calculate:
Sort Z Array
Subroutine
DISTRIBUTION
Calculate:
Z Distribution
Array,
ZVARIANCE
2b


3a
Subroutine
OUTPUT
Input: NAME2
Output: DISREP,MEAN,
VARIANCE,STANDARD DEV,
NCELLS,B,COUNT,ZMEAN,Z,
ZVARIANCE,ZSTANDARD DEV
Subroutine
OUTPUTBOUND
Input: NAME2
Output: DISREP,MEAN
VARIANCE,STANDARD DEV,
NCELLS,B,COUNT,ZMEAN,Z,
ZVARIANCE,ZSTANDARD DEV,
ZLB,ZUB
3b

APPENDIX F

SAMPLE OUTPUT OF THE DISCRETE SIMULATION PROGRAM

RANDOM VARIABLE: 1
DISTRIBUTION: NORMAL
MEAN: 170.000000
VARIANCE: 625.000000
STANDARD DEVIATION: 25.000000
NUMBER OF CELLS : 10

X1 MINIMUM VALUE	CENTER OF GRAVITY	X1 MAXIMUM VALUE
-338.7963616	128.8697140	137.9567811
137.9567811	144.0892129	148.9635820
148.9635820	153.1452715	156.8999533
156.8999533	160.3780729	163.6766683
163.6766683	166.8554752	170.0000025
170.0000025	173.1345248	176.3233317
176.3233317	179.6219271	183.1000467
183.1000467	186.8547285	191.0364180
191.0364180	195.9107871	202.0432189
202.0432189	211.1302860	338.7968616

RANDOM VARIABLE: 2
DISTRIBUTION: NORMAL
MEAN: 29.400000
VARIANCE: 9.000000
STANDARD DEVIATION: 3.000000
NUMBER OF CELLS: 10

X2 MINIMUM VALUE	CENTER OF GRAVITY	X2 MAXIMUM VALUE
-49.6556234	24.4643657	25.5548137
25.5548137	26.2907055	26.8756298
26.8756298	27.3774326	27.8279944
27.8279944	28.2453687	28.6412002
28.6412002	29.0238570	29.4000003
29.4000003	29.7761430	30.1587998
30.1587998	30.5546313	30.9720056
30.9720056	31.4225674	31.9243702
31.9243702	32.5092945	33.2451863
33.2451863	34.3356343	49.6556234

********** Z FUNCTION **********
NUMBER OF UNIQUE Z LOCATIONS: 100
MEAN: 94.25148
VARIANCE: 771.83855
STANDARD DEVIATION: 27.78198

CELL NO.	DISCRETE APPROXIMATION Z	DISCRETE APPROXIMATION DISTRIBUTION	LOWER BOUNDS Z	LOWER BOUNDS DISTRIBUTION	UPPER BOUNDS I	UPPER BOUNDS DISTRIBUTION
1	22.5014176	0.0100000	49.8188562	0.0100000	-445.1651580	0.0100000
2	36.7663186	0.0200000	55.7483597	0.0200000	-436.2811629	0.0200000
3	37.7209165	0.0300000	60.3503526	0.0300000	-426.9347864	0.0300000
4	43.9331503	0.0400000	60.8256572	0.0400000	-421.0052830	0.0400000
5	46.7769751	0.0500000	64.3046270	0.0500000	-416.4032901	0.0500000
6	49.0728865	0.0600000	66.7551607	0.0600000	-412.4490157	0.0600000
7	51.9858175	0.0700000	67.9441992	0.0700000	-408.7894435	0.0700000
8	53.2961557	0.0800000	68.7620284	0.0800000	-405.1622231	0.0800000
9	54.0097765	0.0900000	71.3571536	0.0900000	-401.2616751	0.0900000

10	57.0664044	0.1000000	71.5914196	0.1000000	-396.3968746	0.1000000
11	59.1526492	0.1100000	74.6915319	0.1100000	31.5884847	0.1100000
12	60.4971794	0.1200000	75.3114279	0.1200000	40.4724797	0.1200000
13	60.6793757	0.1300000	75.4919676	0.1300000	42.5952856	0.1300000
14	61.0413761	0.1400000	75.5387434	0.1400000	49.8188562	0.1400000
15	64.2923854	0.1500000	78.9710002	0.1500000	50.5316569	0.1500000
16	64.3939065	0.1600000	79.2935248	0.1600000	51.4792807	0.1600000
17	66.7662254	0.1700000	80.3567680	0.1700000	55.7483597	0.1700000
18	68.2087079	0.1800000	81.4682469	0.1800000	57.3083719	0.1800000
19	68.2746775	0.1900000	81.8620777	0.1900000	59.4156519	0.1900000
20	68.5156546	0.2000000	82.5982205	0.2000000	60.3503526	0.2000000
21	68.6324782	0.2100000	83.2477992	0.2100000	60.8256572	0.2100000
22	72.2861053	0.2200000	86.0702398	0.2200000	63.6317061	0.2200000
23	73.2536307	0.2300000	86.4987686	0.2300000	64.3046270	0.2300000
24	73.3484440	0.2400000	86.9073714	0.2400000	66.1923669	0.2400000
25	74.7620798	0.2500000	87.7915812	0.2500000	66.7551607	0.2500000
26	74.8701848	0.2600000	88.1554069	0.2600000	67.9641992	0.2600000
27	75.4415092	0.2700000	90.0245142	0.2700000	68.7620284	0.2700000
28	75.8983746	0.2800000	90.5345917	0.2800000	69.9550353	0.2800000
29	77.5717132	0.2900000	91.3635690	0.2900000	71.3571536	0.2900000
30	79.6134054	0.3000000	92.3935741	0.3000000	71.5914196	0.3000000
31	80.4864321	0.3100000	93.6340864	0.3100000	72.5157012	0.3100000
32	80.5812454	0.3200000	94.1149104	0.3200000	74.6915319	0.3200000
33	81.0311294	0.3300000	94.4351398	0.3300000	75.3114279	0.3300000
34	81.3421639	0.3400000	94.9621219	0.3400000	75.4919676	0.3400000
35	81.9289115	0.3500000	96.3478484	0.3500000	75.5387434	0.3500000
36	83.8519771	0.3600000	97.3113068	0.3600000	76.7317504	0.3600000
37	84.5045146	0.3700000	98.7169033	0.3700000	78.8390304	0.3700000
38	84.9544332	0.3800000	99.2999402	0.3800000	78.9710002	0.3800000
39	87.0686477	0.3900000	100.0074207	0.3900000	79.2935248	0.3900000
40	87.5185317	0.4000000	100.8916254	0.4000000	80.3567680	0.4000000
41	88.1979611	0.4100000	101.2118548	0.4100000	81.4682469	0.4100000
42	88.5749653	0.4200000	102.6711776	0.4200000	81.8620777	0.4200000
43	88.6694641	0.4300000	102.8984931	0.4300000	82.5982205	0.4300000
44	89.5424907	0.4400000	103.6346410	0.4400000	83.2477992	0.4400000
45	90.0896337	0.4500000	105.4936183	0.4500000	84.6681216	0.4500000
46	91.2919169	0.4600000	106.0766552	0.4600000	85.6157454	0.4600000
47	92.1872346	0.4700000	106.3307499	0.4700000	86.0702398	0.4700000
48	92.9089357	0.4800000	107.5351890	0.4800000	86.4987686	0.4800000
49	93.3376374	0.4900000	108.8279966	0.4900000	86.9073714	0.4900000

50	94.5853634	0.5000000	109.4479926	0.5000000	87.7915812	0.5000000
51	94.7513331	0.5100000	109.9577702	0.5100000	88.1854069	0.5100000
52	95.0523676	0.5200000	112.1375535	0.5300000	90.0245142	0.5200000
53	95.9022655	0.5300000	112.3999895	0.5400000	90.5345917	0.5300000
54	97.5609655	0.5400000	113.1074649	0.5500000	91.3635690	0.5400000
55	98.6746369	0.5500000	113.4299895	0.5600000	92.3935741	0.5500000
56	99.1457424	0.5600000	113.8585183	0.5700000	93.5521166	0.5600000
57	99.3253996	0.5700000	113.9052941	0.5800000	93.6840864	0.5700000
58	100.1409371	0.5800000	116.7346852	0.5900000	94.1149104	0.5800000
59	101.3314172	0.5900000	117.3342639	0.6000000	94.4351398	0.5900000
60	101.9131648	0.6000000	118.7233187	0.6100000	94.9621219	0.6000000
61	102.3326577	0.6100000	119.8367976	0.6200000	95.6749225	0.6100000
62	103.8073217	0.6200000	120.6352333	0.6300000	96.3478486	0.6200000
63	104.0433688	0.6300000	121.0433361	0.6400000	97.3113068	0.6300000
64	104.7619396	0.6400000	123.1443544	0.6600000	98.7169033	0.6400000
65	104.9436365	0.6500000	124.4367905	0.6700000	99.2999402	0.6500000
66	106.3785433	0.6600000	124.6710565	0.6800000	100.0074207	0.6600000
67	106.6282394	0.6700000	125.5000337	0.6900000	100.8916254	0.6700000
68	107.0579010	0.6800000	128.3910648	0.7000000	101.2118548	0.6800000
69	107.9198195	0.6900000	129.5716045	0.7100000	102.6711776	0.6900000
70	108.6587174	0.7000000	131.0807257	0.7300000	102.8984931	0.7000000
71	110.9742234	0.7100000	132.0506371	0.7400000	103.6346410	0.7100000
72	111.2811702	0.7200000	133.4364049	0.7500000	104.5589176	0.7200000
73	111.4310938	0.7300000	135.6778574	0.7600000	105.4936183	0.7300000
74	112.8659461	0.7400000	137.8574407	0.7800000	106.0766552	0.7400000
75	112.9972390	0.7500000	139.5756055	0.7900000	106.3307499	0.7500000
76	115.0516209	0.7600000	144.1907749	0.8100000	107.5351890	0.7600000
77	115.1461197	0.7700000	144.4432059	0.8200000	108.8279966	0.7700000
78	116.1139597	0.7800000	150.5041041	0.8400000	109.4478926	0.7800000
79	118.6638902	0.7900000	157.2808191	0.8600000	109.9579702	0.7900000
80	119.0268906	0.8000000	165.2171904	0.8800000	112.3999895	0.8000000
81	119.1349957	0.8100000	176.2239913	0.9000000	113.1074649	0.8100000
82	119.3346913	0.8200000	250.6589363	0.9100000	113.4299895	0.8200000
83	120.3372283	0.8300000	256.5884402	0.9200000	113.8585183	0.8300000
84	122.3789210	0.8400000	261.1904331	0.9300000	113.9052941	0.8400000
85	124.1076795	0.8500000	265.1447075	0.9400000	116.7346852	0.8500000
86	125.5223980	0.8600000	268.8042797	0.9500000	117.3842639	0.8600000
87	126.1937228	0.8700000	272.4315001	0.9600000	118.7233187	0.8700000
88	126.5174927	0.8800000	276.3320461	0.9700000	119.8367976	0.8800000
89	127.7199435	0.8900000	281.1968486	0.9800000	120.6352333	0.8900000

90	131.3336384	0.9000000	312.9776349	1.0000000	121.0438361	0.9000000
91	131.4349797	0.9100000			124.4367905	0.9100000
92	132.8551373	0.9200000			124.6710565	0.9200000
93	135.5567277	0.9300000			125.5000337	0.9300000
94	135.6735513	0.9400000			129.3910648	0.9400000
95	139.3271784	0.9500000			128.5716045	0.9500000
96	141.9112580	0.9600000			132.0506371	0.9600000
97	142.9394477	0.9700000			133.4364049	0.9700000
98	146.6544736	0.9800000			135.6778574	0.9800000
99	150.8930502	0.9900000			139.5784055	0.9900000
100	157.1307569	1.0000000			144.4432059	1.0000000

APPENDIX G

NOTES ABOUT THE COMPARISON OF THE MONTE CARLO SIMULATION VS. THE DISCRETE SIMULATION

METHODS OF COMPARISON

The Monte Carlo program was written using some of the same subroutines that are used in the discrete simulation program. These subroutines are: DISTRIBUTION, FUNCTION, OUTPUT, and SORT. Subroutine BOOK is also used, however, a few lines of code had to be modified for the generation of the random numbers. The random numbers are generated using a normal random number generator provided by Harris Computer Systems.

Because the two methods are slightly different, a method of comparison had to be devised. Three methods were possible:

1. Use the same number of points to represent the random variables' density functions.
2. Use the same number of possible combinations of outcomes to represent the joint random variable's distribution function.
3. Use the same number of unique outcomes to represent the joint random variable's distribution function.

Method 1 was thought to give a distinct advantage to the discrete approximation program, while method 2 would give an advantage to the Monte Carlo simulation. Method 3 was chosen to be used as the method of comparison because it is somewhat of a compromise between methods 1 and 2. However, this method does give the discrete simulation a slight advantage over the Monte Carlo method for the example discussed in Chapter 5.

The discrete simulation program was run first each time the two methods were compared. After each discrete approximation, the number of unique outcomes m, was noted. The joint distribution function was then approximated using the Monte Carlo program by generating m pairs of random variables X and Y.

CPU TIME COMPARISON

Figures 5.18 and 5.19 were plotted to give an overall comparison of the computer processing unit time for both programs. A binary sorting routine was used for both programs. Since the outcomes generated by the discrete method are somewhat ordered, the discrete values are sorted quicker than the data that is created randomly from the Monte Carlo program. It should be noted that

these times are largely dependent on the efficiency of the sorting routine. Quicker times could be achieved by using a faster method of sorting the outcomes. For example, a quick sort subroutine would decrease the run time of the Monte Carlo program since this method sorts random numbers faster than ordered numbers. Also, a heap sort could be used to decrease the CPU time for the ordered discrete values.

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VITA

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Master of Science

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THE APPROXIMATION OF JOINT DISTRIBUTION FUNCTIONS FOR APPLICATION IN PROBABILISTIC MECHANICAL DESIGN

by

BRUCE EUGENE SWANSON

B.S., Kansas State University, 1985

AN ABSTRACT OF A MASTER'S THESIS

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ABSTRACT

In this study, a discrete simulation technique is developed to approximate the joint probability distribution functions of complex algebraic expressions containing several independent random variables. The discrete simulation package is written in Fortran on a Harris H-800 Super Mini-Computer.

The discrete approximation is compared to the traditional Monte Carlo technique for a joint probability distribution function that can be solved exactly. The accuracy of the discrete simulation is found to be better than the Monte Carlo simulation.

A cantilever I-beam is analyzed to illustrate how the discrete simulation package can be used by design engineers in a variety of engineering applications.

A bounding program is also discussed that bounds the exact joint probability distribution function of an algebraic expression. Although a formal proof has not been developed, the results of several different examples show considerable promise for further study.